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S3, Mathematics , Unit 1:PROBLEMS ON SETS

LESSON 1: Review of union, intersection and complement of sets and Representation of problems using a Venn diagram

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Unit 1: PROBLEMS ON SETS
1.1 Review of union, intersection and complement of sets

- The set of common elements which appear in two or more sets is called **the intersection of the sets**, It's symbol is ∩, Can also presented as **"and"** in word statement. For example, **"sets A and B" means A∩B**.
- When the elements of two or more sets are put together to form a set, the set formed is known as **union of sets**, its symbol is ∪, is also represented by "or" in word statement. For example, **"Sets A or B"** means **A∪B** that is the union of sets A and B.
- **Complement of a set** is the set of all elements in the universal set that are not members of a given set. The complement of set **A** is denoted by **A**'.
- A universal set contains all the subsets under consideration. It is denoted by the symbol ε.

1.1 Review of union, intersection and complement of sets

- **Example 1.1**

Given the following sets A = {*a, b, c, d, e, f*}
and B = {*a, b, c, h, i,*} find:
(i) $(A \cap B)$ (ii) $(A \cup B)$

- **Solution**

(i) $(A \cap B)$ = {*a, b, c*}

(ii) $A \cup B$ = {*a, b, c, d, e, f*} ∪ {*a, b, c, h, i,*}
= {*a, b, c, d, e, f, h,* i}

- **Example 1.2**

Given A = {*1*, *2*, *3*, *4*, *5*}, B = {*2*, *4*, *6*} and C = {*1*, *3*, *5*, *7*, *9*}, answer the questions below about the sets A, B and C.

(a) List the set $A \cap B$. **(b)** Write down n(*A*). **(c)** List the set $A \cup B$. **(d)** List the set $A \cup B \cup C$.

1.1 Review of union, intersection and complement of sets

- **Solution**

(a) $A \cap B = \{2,4\}$

(b) $n(A) = 5$

(c) $A \cup B = \{1, 2, 3, 4, 5, 6\}$

(d) $A \cup B \cup C = \{1,2,3,4,5,6,7,9\}$

1.2 Representation of problems using a Venn diagram
1.2.1 Venn diagrams involving two sets

- Some important facts like
    - "intersection",
    - "union" and
    - "complement" should be well considered and represented when drawing Venn diagrams.
- Consider two intersecting sets A and B such that A = {a, b, c, d, e, f} and B = {a, b, c, d, g, i, j, k, l}. We represent the two sets in a set diagram as
- shown in **Fig 1.1 below**.

1.2.1 Venn diagrams involving two sets

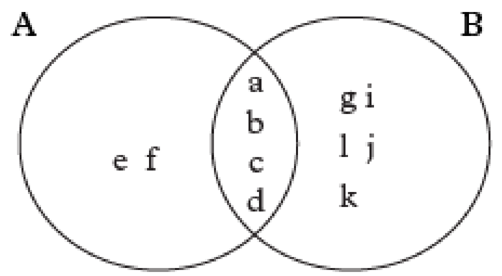


Fig 1.1

- The union of sets A and B is given by the number of elements.
- $\boldsymbol{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$
  In the Venn diagram in Fig 1.1,
  $n(A) = 6$, $n(B) = 9$ and $n(A \cap B)=4$
  $\Rightarrow n(A \cup B)= 6 + 9 - 4 = 11$

1.2.1 Venn diagrams involving two sets

Example 1.3

- Fig. 1.2 shows the marks out of 15 scored by a number of Senior 3 students in groups C and D.

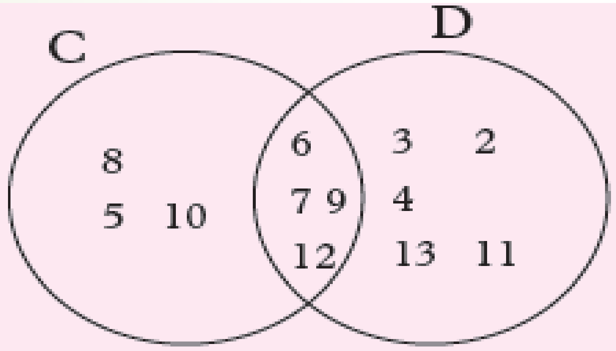


Fig 1.2

Determine the total number of Senior 3 students in the two groups.

1.2.1 Venn diagrams involving two sets

- **Solution**
- $n(C \cup D) = n(C) + n(D) - n(C \cap D) = 7 + 9 - 4 = 16 - 4 = 12$
- **Example 1.4**

A survey involving 120 people about their preferred breakfast showed that;55 drink milk at breakfast, 40 drink juice at breakfast and 25 drink both milk and juice at breakfast.

(a) Represent the information on a Venn diagram.

(b) Calculate the following:

(i) Number of people who take milk only.

(ii) Number of people who take neither milk nor juice.

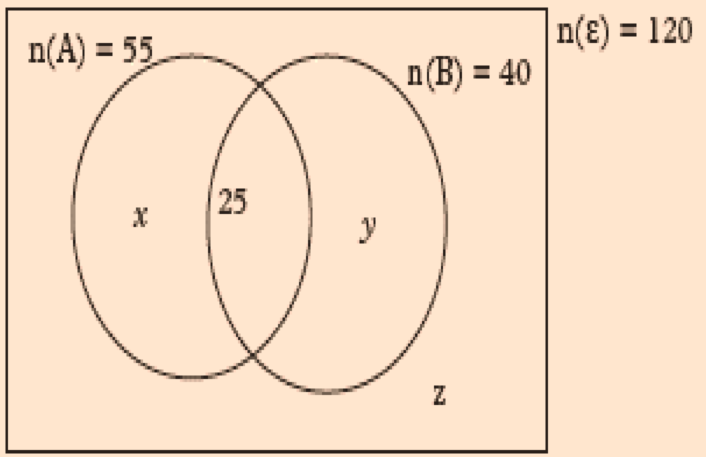


Fig. 1.3

1.2.1 Venn diagrams involving two sets

Solution

- (a) Let A be the set of those who drink milk and B be the set of those who drink juice, x be the number of those who drink milk only, y be the number of those who drink juice only and z represents number of those who did not take any.
  By expressing data in set notation;
  $n(A) = 55$, $n(B) = 40$, $n(A \cap B') = x$,
  $n(A \cap B) = 25$, $n(A' \cap B) = y$.
  $n(\varepsilon) = 120$.

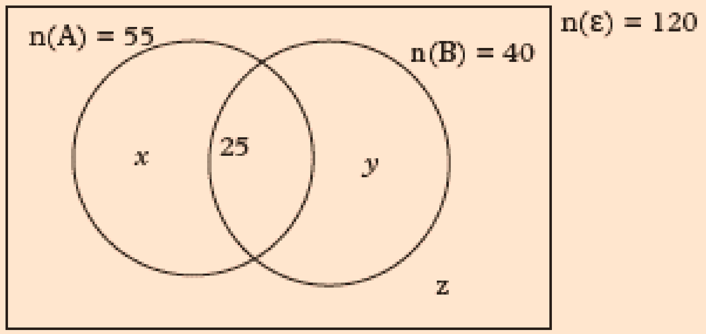


Fig. 1.3

1.2.1 Venn diagrams involving two sets

(b) (i) We are required to find the number of those who take milk only.
$x = 55 – 25 = 30$ So, 30 people take milk only.

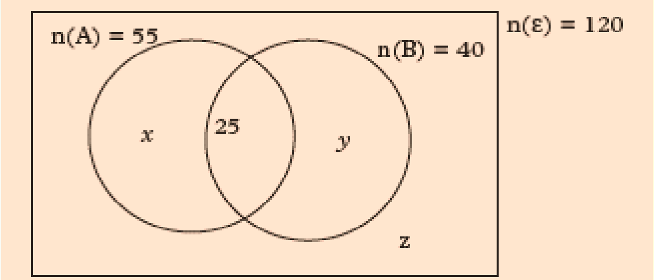


Fig. 1.3

(ii) To find the value of z;
$30 + 25 + 15 + z = 120.$
$z = 120 – (30+ 15 + 25).$
$z = 120 – 70 \Rightarrow z = 50.$
So, 50 people take neither eggs nor juice for breakfast.

- **Example 1.5**
  In a class of 20 pupils, 12 take Art (A) and 10 take Chemistry (C). The number that take none is half the number that take both.

(a) Represent the information on a Venn diagram.

(b) Use the Venn diagram to determine the number that take:
(i) Both (ii) None

1.2.1 Venn diagrams involving two sets

- **Solution**

We first extract the data and represent it in set notation.

$n(\varepsilon) = 20, n(A) = 12, n(C) = 10$

Let $n(A \cap C) = y$ so, $n(A \cup C)' = \frac{1}{2}y$

(a)

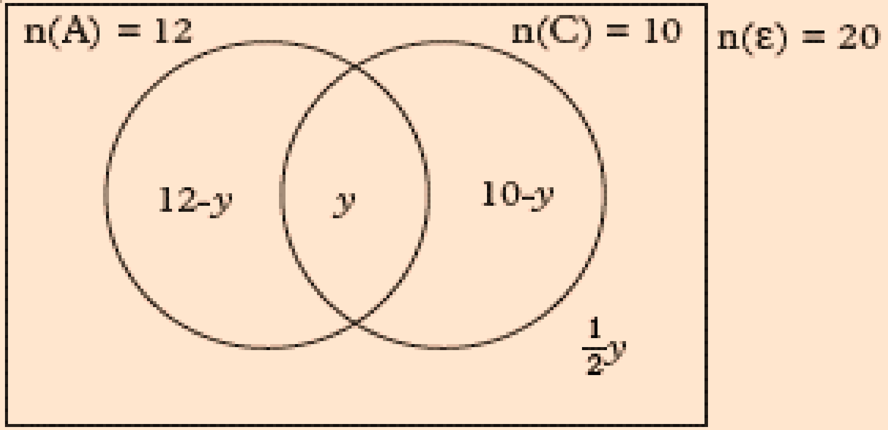


Fig. 1.4

1.2.1 Venn diagrams involving two sets

(b) By solving for the value of y;
$12 - y + y + 10 - y + 1/2y = 20$

So, $22 - 1/2y = 20$
Collecting like terms together,
we get, $-1/2y = 20 - 22$
$-1/2y = -2$
$-y = -4$

$y = 4$
(i) Those who take both are equal to 4.
(ii) Those who take none $= 1/2 \times 4 = 2$

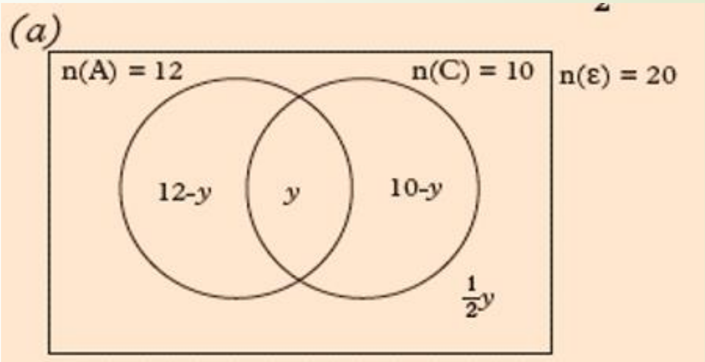


Fig. 1.4

1.2.2 Venn diagrams involving three sets

- without a Venn diagram, the problems involving three or more sets become complicated to handle.
  A Venn diagram makes the problem easier because we can represent the data
  extracted in each region and then calculate the values required. Consider the venn diagram shown in Fig. 1.6 showing the numbers of students who take the foreign languages **German (G**), **Spanish(S**) and **French(F)** in a college.

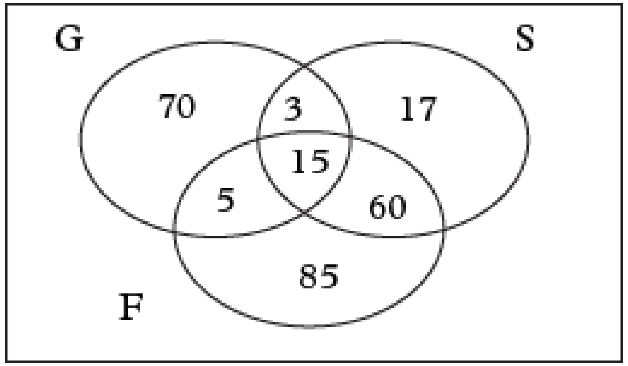


Fig. 1.6

1.2.2 Venn diagrams involving three sets

- The total number of students taking languages is given by the union of the three sets as shown by the following formula
- $n(G \cup S \cup F) = n(G) + n(S) + n(F) - \{n(G \cap S) + n(G \cap F) + (S \cap F) + n(G \cap S \cap F)\}$

From the Venn diagram in Fig 1.6,

$n(G \cup S \cup F) = 93 + 95 + 165 - (18 + 20 + 20 + 75) + 15$

$= 353 - 113 + 15$

$= 255$

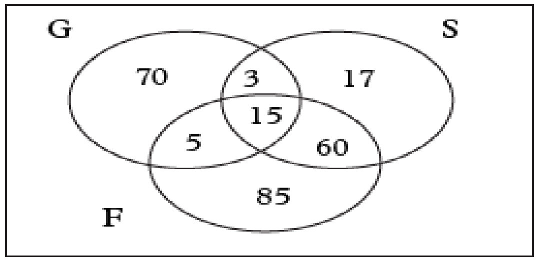


Fig. 1.6

1.2.2 Venn diagrams involving three sets

- **Example 1.6**

The students in Senior 3 class did a survey on their names regarding whether they contained the letters B, C and D. The following Venn diagram shows the results of the survey in terms of the number of names in each category:

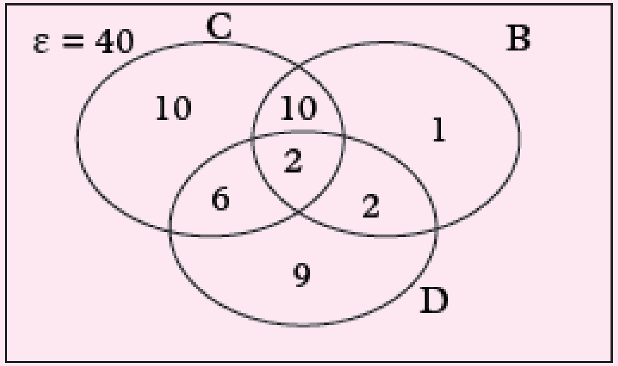


Fig. 1.7

1.2.2 Venn diagrams involving three sets

- Use the Venn diagram to determine the number student's names that contained in;

(a) All the three letters

(b) Letter D

(c) Letters B and D but not C

(d) Only two of the letters

(e) The total number of students

Solution:

(a) All the three letters
$n(B \cap C \cap D) = 2$
(b) $n(D) = 9 + 6 + 2 + 2 = 19$

(c) Letters B and D but not C
$n(B \cap D) - n(B \cap C \cap D) = 4 - 2 = 2$

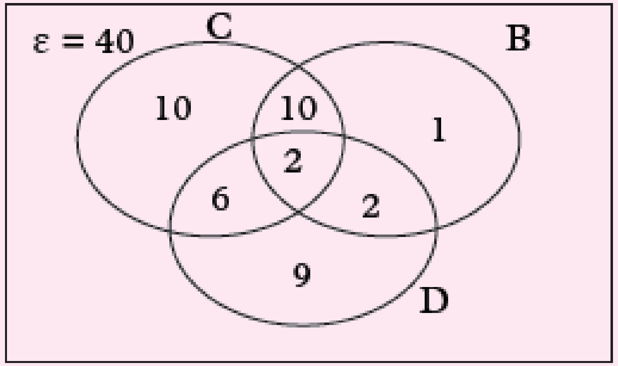


Fig. 1.7

1.2.2 Venn diagrams involving three sets

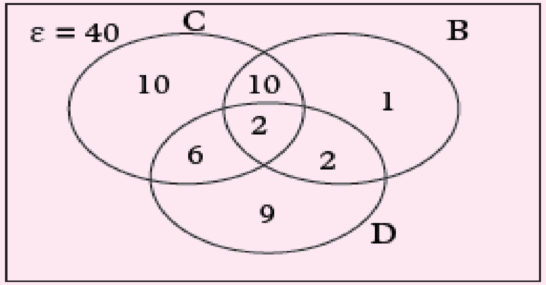


Fig. 1.7

(d) Only two of the letters
= 6 +10 + 2 =18

(e) The total number of students
n(B∪C∪D)= n(B) + n(C) + (C)
-{n(B∩C) + n(B∩D)+n(C∩D)} +n(B∩C∩D)
= 15 + 28 + 19 -{12 + 4 + 8} + 2 = 62 - 24 + 2= 40 Students

- **Example 1.7**
A group of 40 tourists arrived in Rwanda and visited Akagera National park (A), Nyungwe forests (N) and Virunga mountains (V). Results showed that 33 visited Akagera, 21 visited Nyungwe and 23 visited Virunga. 18 visited both Akagera and Nyungwe, 10 visited both Nyungwe and Virunga, and 17 visited both Akagera and Virunga. All tourists visited at least one of the places.
(a) Represent the information on a Venn diagram.

(b) Find the number of tourists that visited:

(i) Akagera only.

(ii) Did not visit Nyungwe.

1.2.2 Venn diagrams involving three sets

- **Solution**

$n(\varepsilon) = 40$.
$n(A) = 33$, $n(N) = 21$, $n(V) = 23$.
$n(A \cap N) = 18$, $n(N \cap V) = 10$,
$n(A \cap V) = 17$.
Let $n(A \cap N \cap V) = y$

(a)

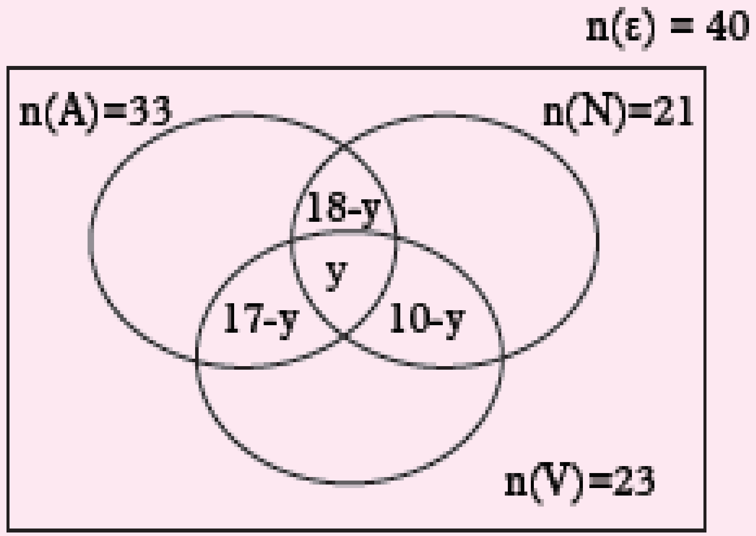


Fig. 1.8

1.2.2 Venn diagrams involving three sets

- $n(A)$ *only* $= n(A \cap N' \cap V')$.

$n(A \cap N' \cap V') = 33 - (18 - y + y + 17 - y)$.

$n(A \cap N' \cap V') = 33 - 35 + y = y - 2$.

$n(A \cap N' \cap V') = y - 2$.

- $n(N)$ *only* $= n(A' \cap N \cap V')$.

$n(A' \cap N \cap V') = 21 - (18 - y + y + 10 - y)$.

$n(A' \cap N \cap V') = 21 - 28 + y = -7 + y$.

$n(A' \cap N \cap V') = y - 7$.

- $n(V)$ *only* $= n(A' \cap N' \cap V)$.

$n(A' \cap N' \cap V) = 23 - (17 - y + y + 10 - y)$.

$n(A' \cap N' \cap V) = 23 - 27 + y$.

$n(A' \cap N' \cap V) = y - 4$.

The Venn diagram in Fig. 1.9 shows the data in specific regions.

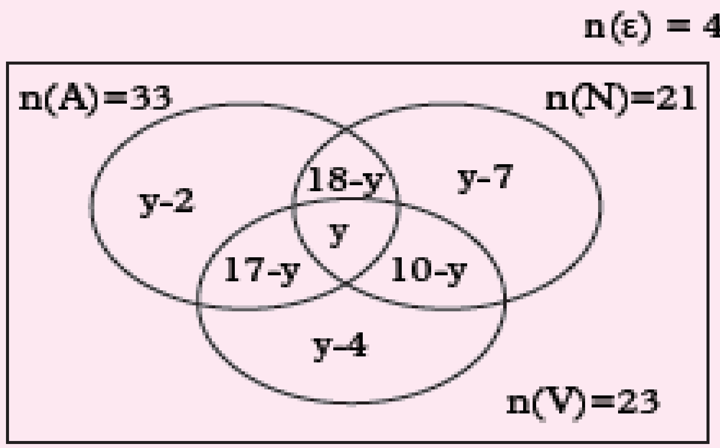


Fig. 1.9

1.2.2 Venn diagrams involving three sets

$y - 2 + 18 - y + y - 7 + 17 - y + y + 10 - y + y - 4 = 40$

$y + 32 = 40.$

$y = 8.$

- (b) (i) Those who visited Akagera only are $y - 2 = 8 - 2 = 6$.
- (ii) Those who did not visit Nyungwe are $y - 4 + 17 - y + y - 2$
  $= 8 - 4 + 17 - 8 + 8 - 2$
  $= 19$

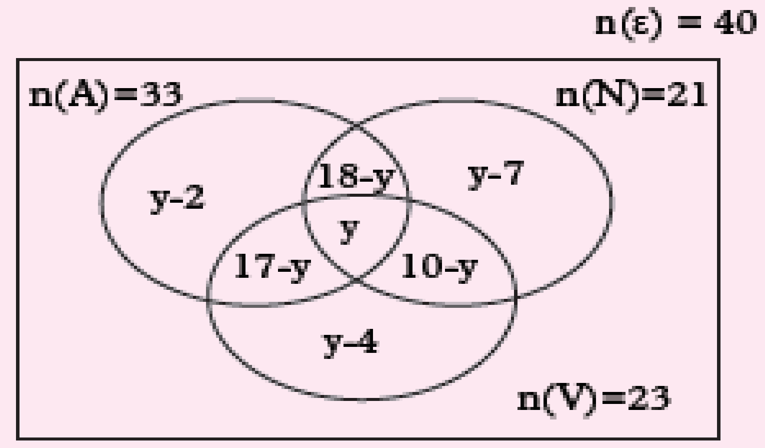


Fig. 1.9

Example 1.8

The Venn diagram below shows the allocation of the members of the Board of Directors of a school in three different committees;

Academic (A), Production (P) and Finance (F).

1.2.2 Venn diagrams involving three sets

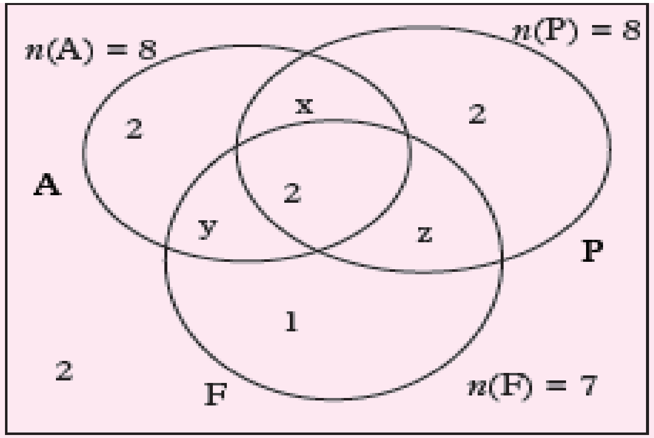


Fig. 1.10

(a) Determine the values of x, y and z.

(b) What is the total number of members in the Board of Directors?

(c) Find the number of those who are not members of the academic committee.

(d) How many belong to at least two committees?

1.2.2 Venn diagrams involving three sets

- **Solution**
- (a) For Academic,
  $2 + x + 2 + y = 8$. $x + y = 8 - 4$. $x + y = 4$........(i)

  For Production,
  $2 + x + 2 + z = 8$. $x + z = 8 – 4$. $x + z = 4$...............(ii)
  For Finance,
- $1 + y + 2 + z = 7$.
  $y + z = 7 – 3$. $y + z = 4$..............(iii)
  Make x the subject in equation (i)
  $x = 4 — y$........(iv)
  Substitute equation (iv) into (ii)
  $4 – y + z = 4$.

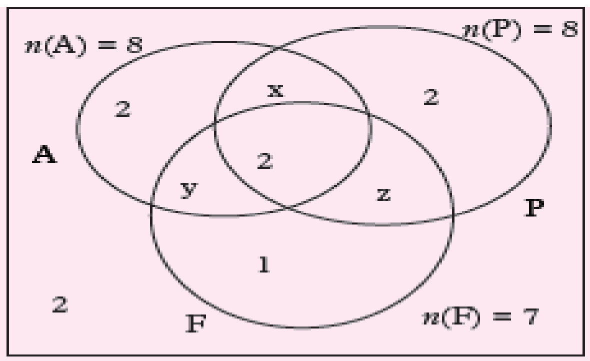


Fig. 1.10

1.2.2 Venn diagrams involving three sets

$-y + z = 0$........(v)
Solving (iii) and (v) simultaneously

$y + z = 4$

$-y + z = 0$

$2z = 4 \Rightarrow z = 2$

So, $y = 4 - z = 4 - 2 = 2$

$\therefore y = 2.$

$x + y = 4$ and so $x = 4 - y = 4 - 2 = 2.$

$\therefore x = 2.$

(b) Total number of members are

$2 + 2 + 2 + 2 + 2 + 2 + 1 + 2 = 15$

(c) Those who are not members of academic

committee are: $2 + 1 + z + 2$

$= 2 + 1 + 2 + 2 = 7$

(d) Those who belong to at least two committees are: $y + 2 + x + z$

$= 2 + 2 + 2 + 2 = 8$

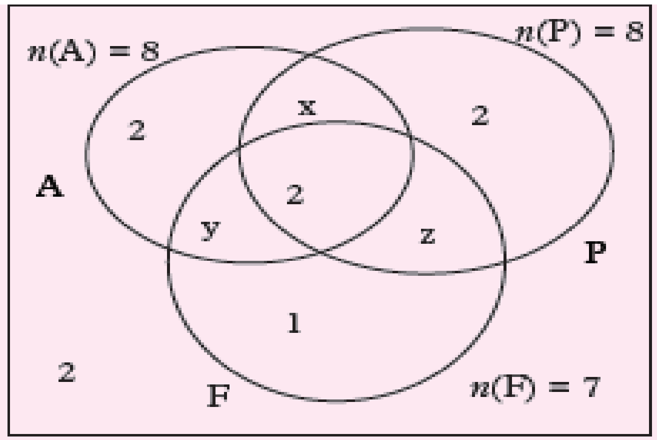
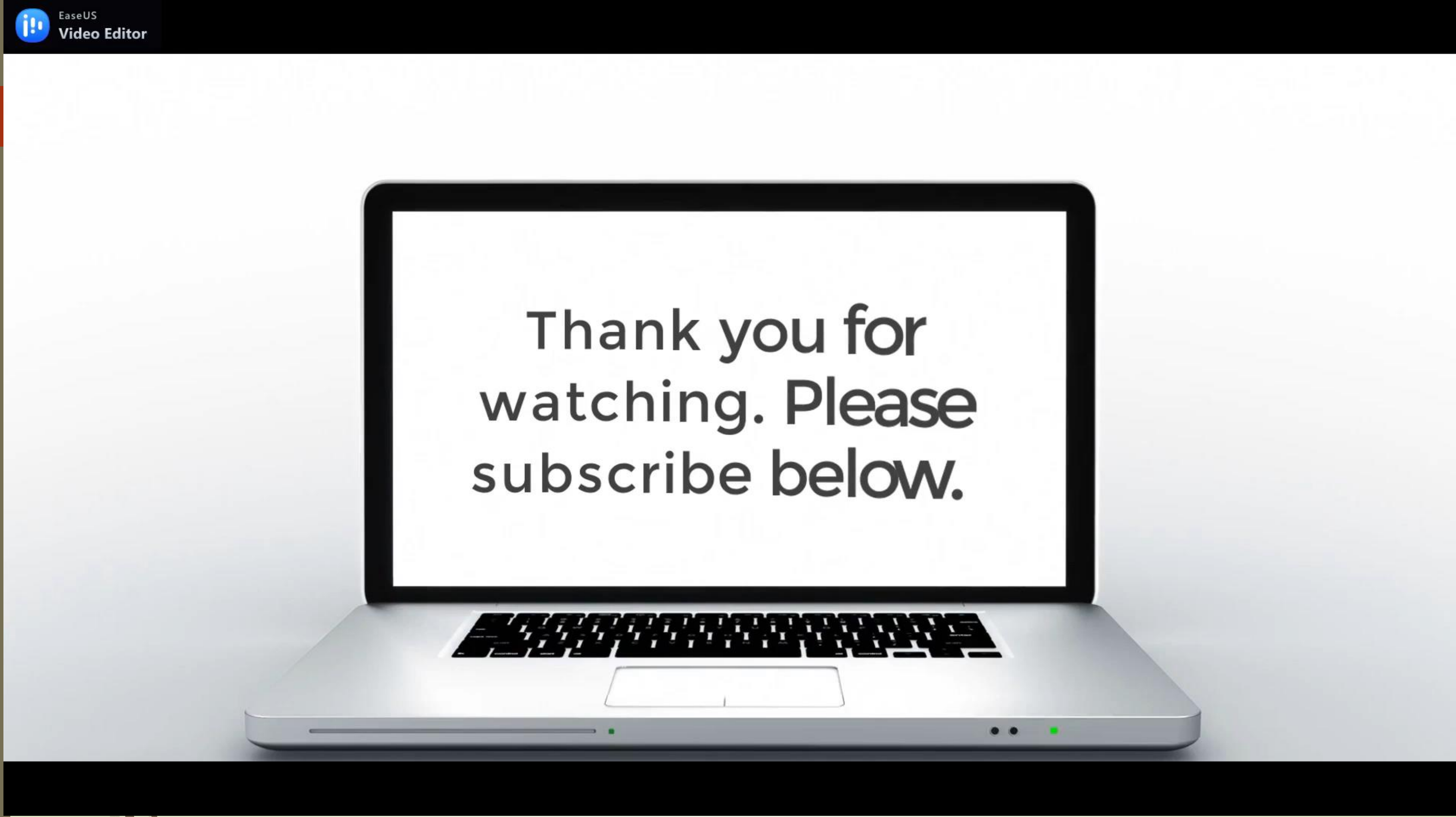


Fig. 1.10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