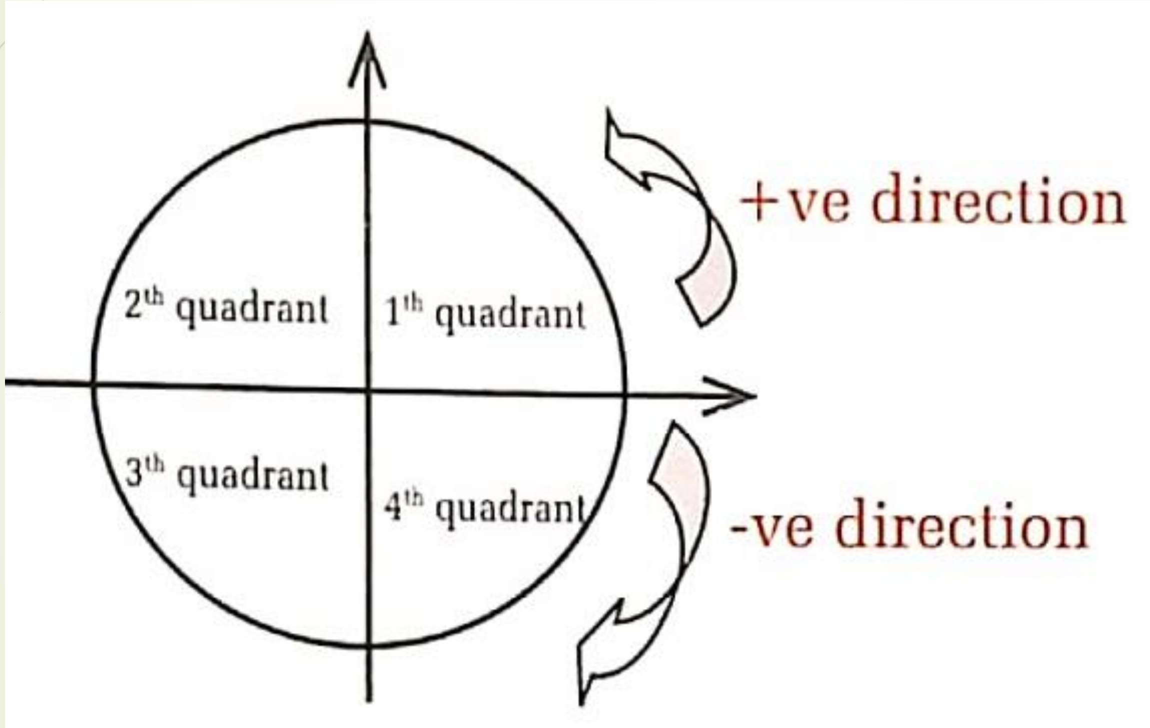
UNIT 1 : **Fundamentals of trigonometry**

1.1 Trigonometric concepts

- The word **trigonometry** is derived from two Greek words: **Trigon**, which means **triangle**, and **metric**, which means **measure**.
- **Trigonometry** is the study of how the sides and angles of a triangle are related to each other.
- **Angles** are named according to where their terminal side lies,
- The x-axis and y-axis divide a plane into **four quadrants** as follow.

1.1 Trigonometric concepts



Angle is positive if rotated in a counterclockwise direction and negative when rotated clockwise.

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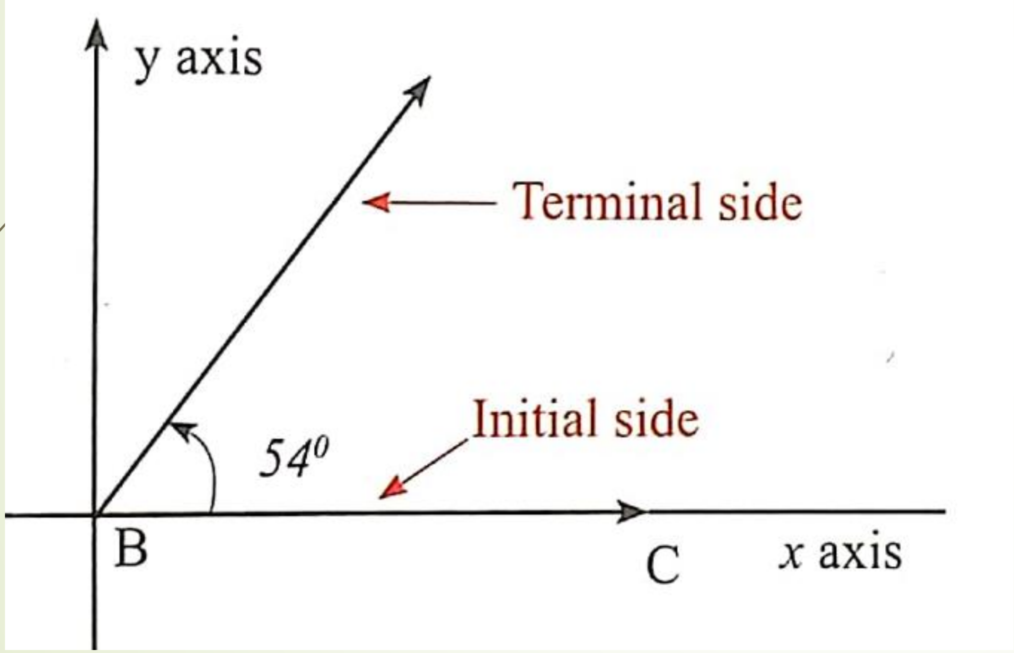
- A **rotation angle** is formed by an initial side through an angle, about a fixed point called vertex, to terminal position called **terminal side**.
- **Example 1:**



Angles in standard position that have a common terminal side are called **co-terminal angles**

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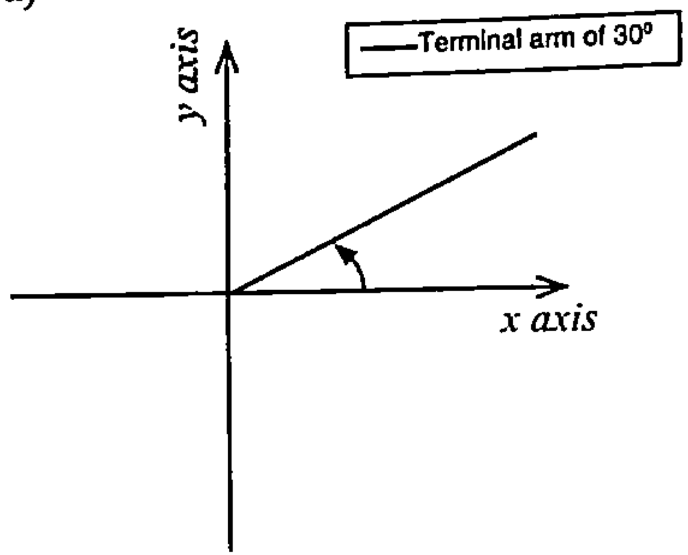
- **Example 2:**

Draw each of the following angles in standard position and which of these angles is co-terminal to 30°?

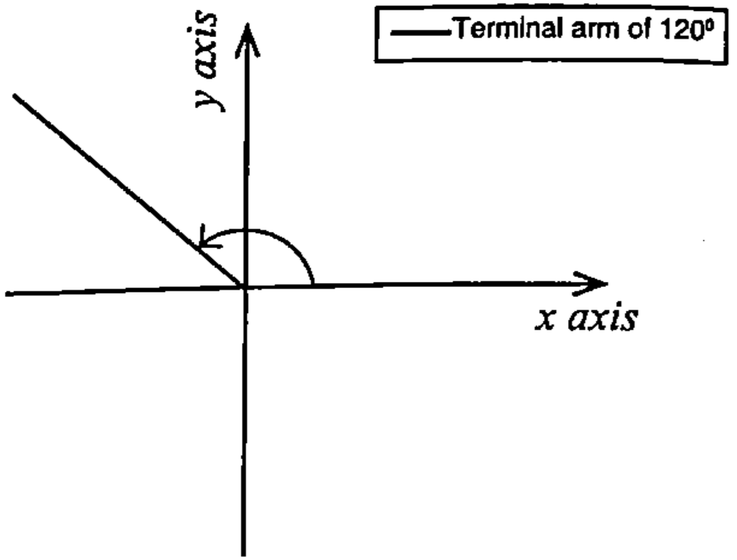
a) 30° b) 120° c) -230° d) 750° e) -330°

Solution:

a)



b)



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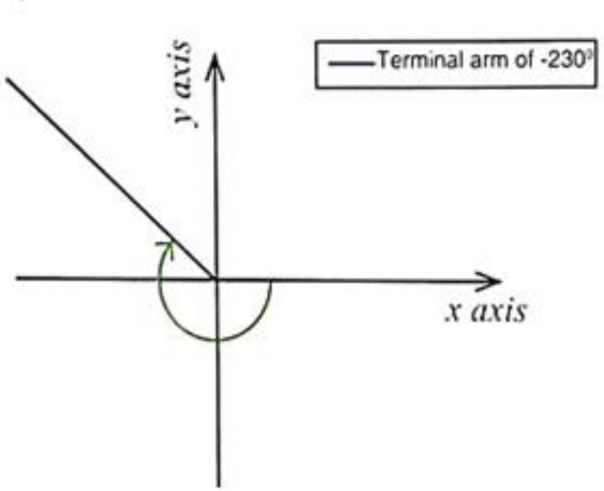
c)



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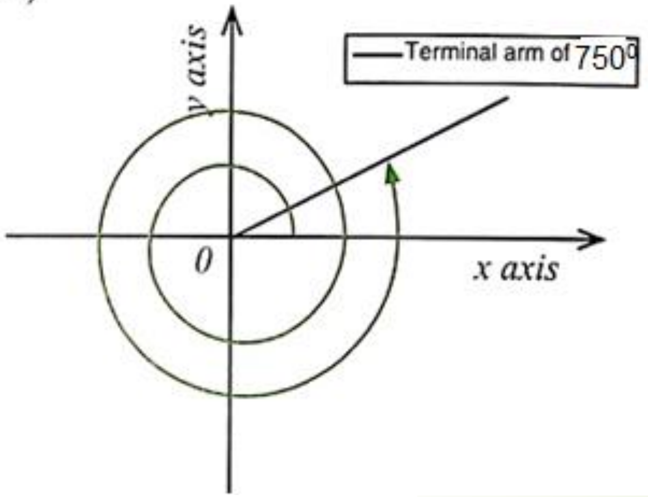

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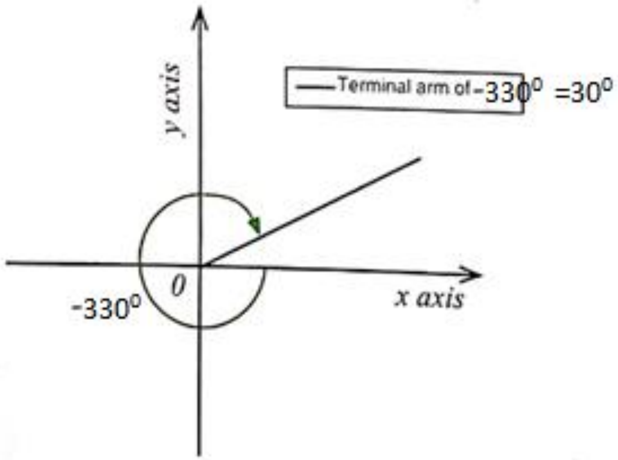


750° and -330° are co-terminal to 30°

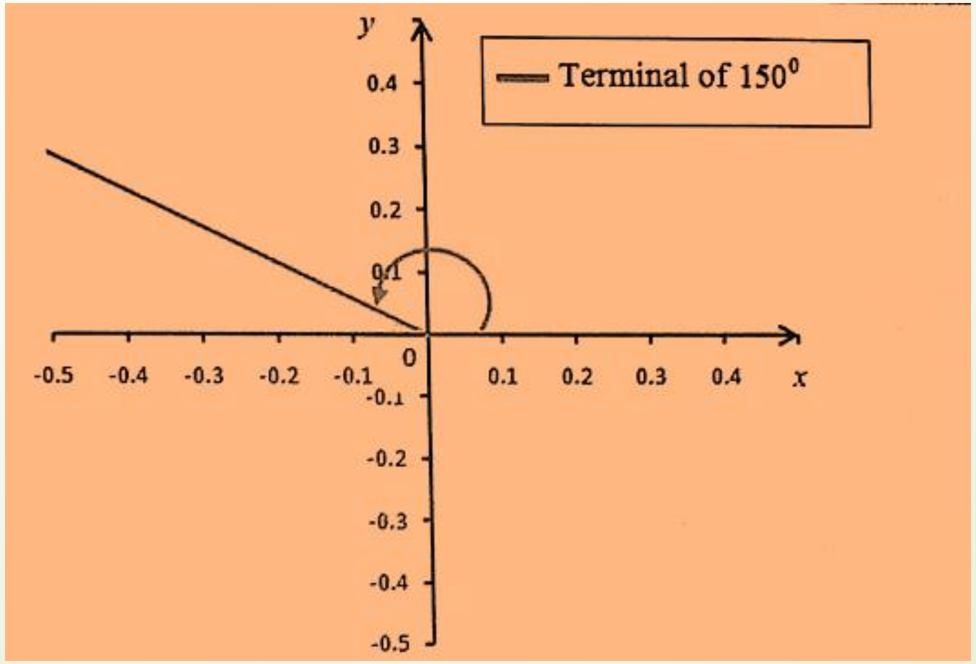
1.1 Trigonometric concepts

- **Example 3:**

Draw each of the following angles in standard position and indicate in which quadrant the terminal side is.

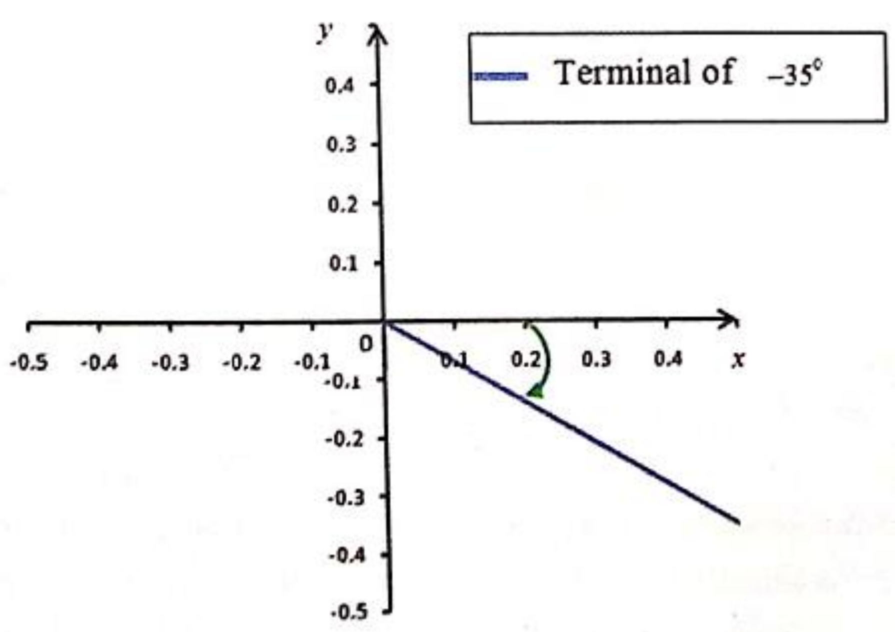
a) 150° b) -35° c) 210°

- **Solution:**

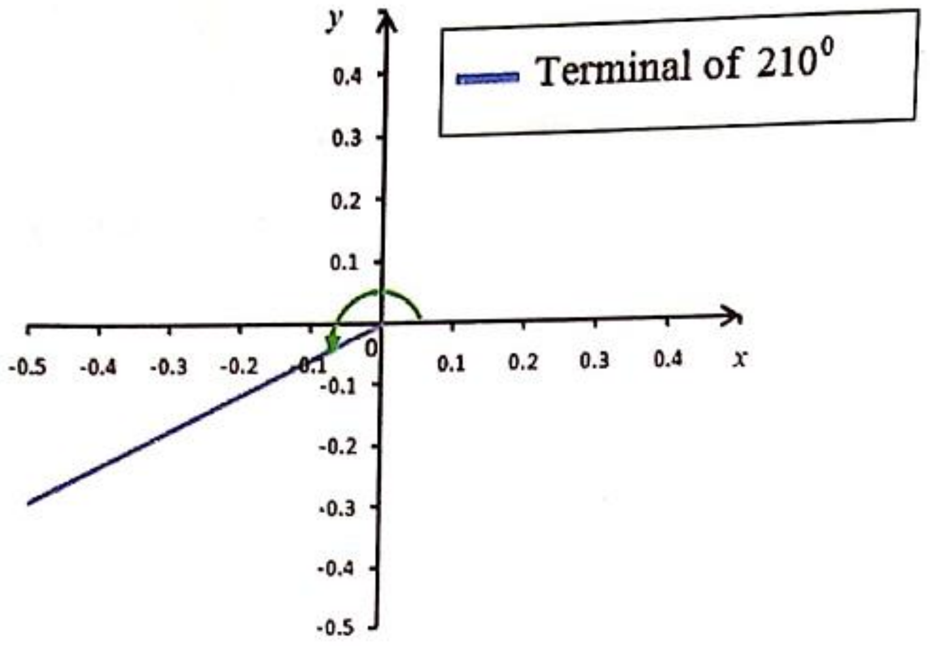


1.1 Trigonometric concepts

150° lies in 2^{nd} quadrant



-35° lies in 4^{th} quadrant



210° lies in 3^{rd} quadrant

1.1 Trigonometric concepts

- Measure of an angle

The amount we rotate the angle is called the measure of the angle and is measured in the following units:

- a) Sexagesimal system

Unit is **degree** (written with a superscript $^\circ$). One degree (1°) is $\frac{1}{90}$ of the right angle. In angular measure, the degree is subdivided into **minutes** and **seconds** ($'$ and $''$). $1^\circ = 60'$, $1' = 60''$.

1.1 Trigonometric concepts

- Example 4:

The angle which measures 12 degrees, 35 minutes and 15 seconds will be noted by $12^{\circ}35'15''$.

- Degrees, minutes second:

Degrees (°), minutes (‘), seconds (“) to decimal degrees and vice versa

- Let d represents the integer degrees, dd represents the integer part of the decimal degree, m represents minutes and s represents seconds. Then:

$$d = integer(dd)$$

$$m = integer((dd-d)\times 60)$$

$$s = \left(dd - d - \frac{m}{60}\right) \times 3600$$

1.1 Trigonometric concepts

- For angle with d integer degrees, m minutes and s seconds:

  $d^{\circ} m' s''$

  The decimal degrees (dd) is equal to

  $$dd = d + \frac{m}{60} + \frac{s}{3600}$$

- **Example 5:**

Convert 30.263888889° to $d^{\circ} m' s''$ system

- **Solution:**

$$d = integer\left(30.263888889^{\circ}\right) = 30^{\circ}$$

$$m = integer\left(\left(dd - d\right)\times 60\right) = integer\left(\left(30.263888889^{\circ} - 30^{\circ}\right)\times 60\right) = 15'$$

$$s = \left(dd - d - \frac{m}{60}\right)\times 3600 = \left(30.263888889^{\circ} - 30^{\circ} - \frac{15}{60}\right)\times 3600 = 50''$$

So

$30.263888889^{\circ} = 30^{\circ}\ 15'\ 50''$

1.1 Trigonometric concepts

- **Example 6:**

Convert 30 degrees 15 minutes and 50 seconds angle to decimal degrees

- **Solution:**

30º 15’ 50”

The decimal degrees dd is equal to:

$$dd = d + \frac{m}{60} + \frac{s}{3600}$$

$$= 30^\circ + \frac{15'}{60} + \frac{50''}{3600}$$

$$= 30.263888889^\circ$$

1.1 Trigonometric concepts

- b) Centesimal system

Unit is **grade**. One grade is equal to $\frac{1}{100}$ of the right angle and is subdivided into

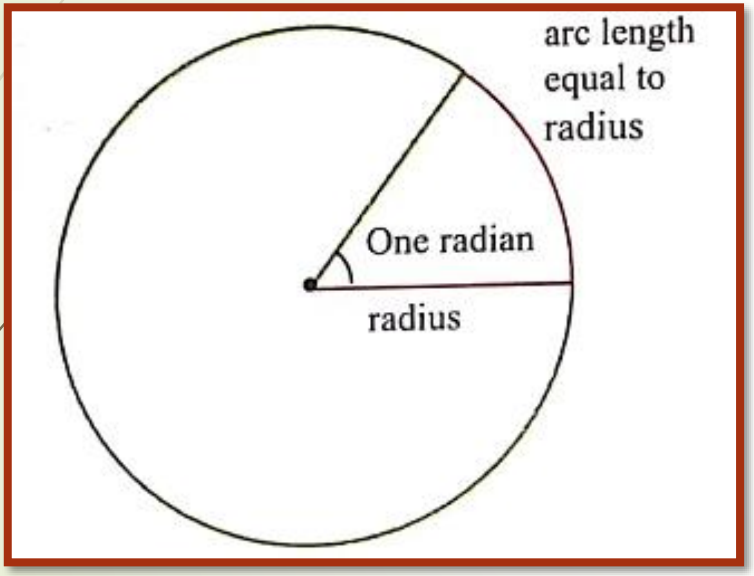
decigrade: $\frac{1}{10}$ grades, **centigrade**: $\frac{1}{100}$ grades and **milligrade**: $\frac{1}{1000}$ grades

- Example 7:

An angle which measures 82 grades, 7 decigrades, 2 centigrades and 5 milligrades will be noted by $82^{G},725$.

1.1 Trigonometric concepts

- c) Radian



- When a central angle intercepts an arc that has the same length as a radius of the circle, the measure of the angle is defined to de one radian.
- Like degrees, radian measures the amount of the rotation from the initial side to the terminal side of an angle.

1.1 Trigonometric concepts

- Proportions between three units

Since 2π radians $= 360$ degrees,

then π radians $= 180$ degrees

- 360 degrees=400grades, 180degrees=200grages

$$\frac{D}{180} = \frac{R}{\pi} = \frac{G}{200}$$

where D stands for degree, R for radians, G for grades and $\pi = 3.14...$

This relation can be split into 3 relations:

$$\frac{D}{180} = \frac{R}{\pi}, \quad \frac{D}{180} = \frac{G}{200} \text{ and } \frac{R}{\pi} = \frac{G}{200}$$

1.1 Trigonometric concepts

- **Example 8**:

Convert 90° to radians and grades

- **Solution:**

$$\frac{D}{180}=\frac{R}{\pi}\Leftrightarrow\frac{90}{180}=\frac{R}{\pi}$$

$$\Leftrightarrow R=\frac{90\pi}{180}=\frac{\pi}{2}\quad or\ R=1.57$$

$$\frac{D}{180}=\frac{G}{200}\Leftrightarrow\frac{90}{180}=\frac{G}{200}$$

$$\Leftrightarrow G=\frac{90\times200}{180}=100$$

Thus, $90^\circ=\frac{\pi}{2}\,radians\ or\ 1.57\,radians$ and $90^0 = 100 grades$

1.1 Trigonometric concepts

- **Example 9**:

Convert 20 grades to radians and degrees

- **Solution:**

$$\frac{R}{\pi} = \frac{G}{200} \Leftrightarrow \frac{R}{\pi} = \frac{20}{200}$$

$$\Leftrightarrow R = \frac{20\pi}{200} = \frac{\pi}{10} \quad or\ R = 0.314$$

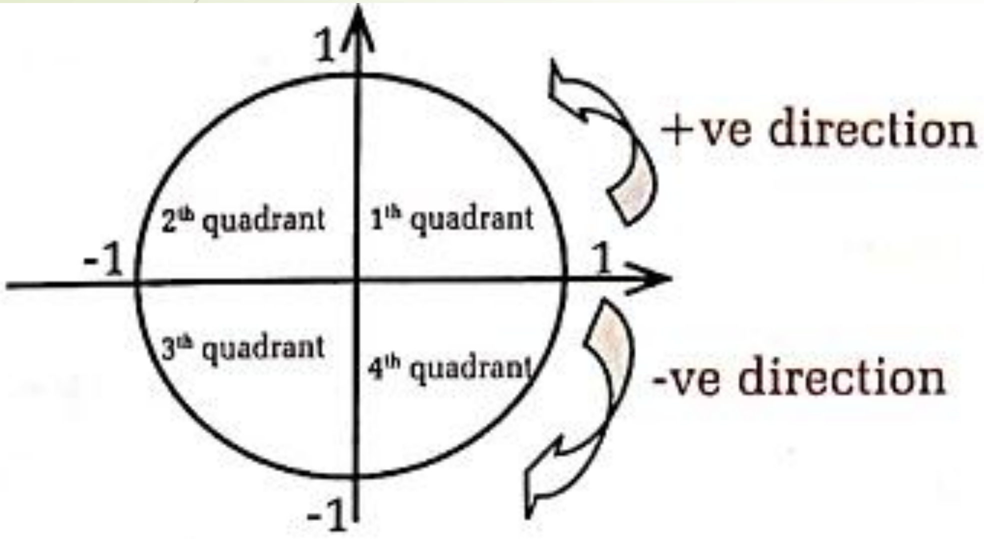
$$\frac{D}{180} = \frac{G}{200} \Leftrightarrow \frac{D}{180} = \frac{20}{200}$$

$$\Leftrightarrow D = \frac{20 \times 180}{200} = 18$$

Thus, $20\,grades = 18^0$ and $20\,grades = \dfrac{\pi}{10}\,radians\ or\ 0.314\,radians$

1.1 Trigonometric concepts

- **Unit circle**



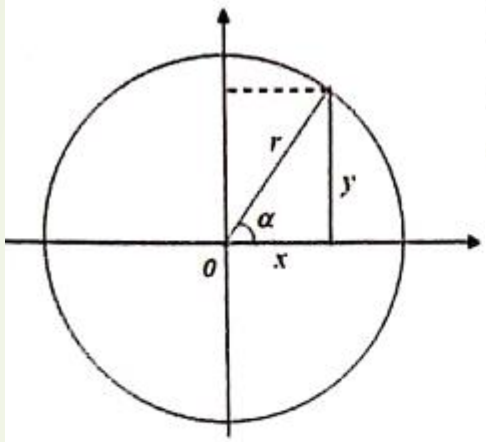
A **unit circle** is a circle of radius one centered at the origin (0,0) in the Cartesian coordinate system in the Euclidean plane.

In the unit circle, the coordinate axes delimit four quadrants that are numbered in an anticlockwise direction. Each quadrant measures 90 degrees, means that the entire circle measures 360 degrees or 2π radians.

1.1 Trigonometric concepts

- **Trigonometric ratios of acute angles**

Consider the following circle with radius r



In the right angled triangle, we define the following six ratios:

The three primary trigonometric values

- The ratio $\frac{x}{r}$ is called **cosine** of the angle α, noted $\cos\alpha$.
- The ratio $\frac{y}{r}$ is called **sine** of the angle α, noted $\sin\alpha$.
- The ratio $\frac{y}{x}$ is called **tangent** of the angle α, noted $\tan\alpha$.

The three reciprocal trigonometric values

- **secant** of the angle α, noted $\sec\alpha$ is the ratio $\frac{r}{x}$
- **cosecant** of the angle α, noted $\csc\alpha$ is the ratio $\frac{r}{y}$
- **cotangent** of the angle α, noted $\cot\alpha$ is the ratio $\frac{x}{y}$

Observing the triangle in the above circle, we see that r is the hypotenuse, x is the adjacent side and y is the opposite side.

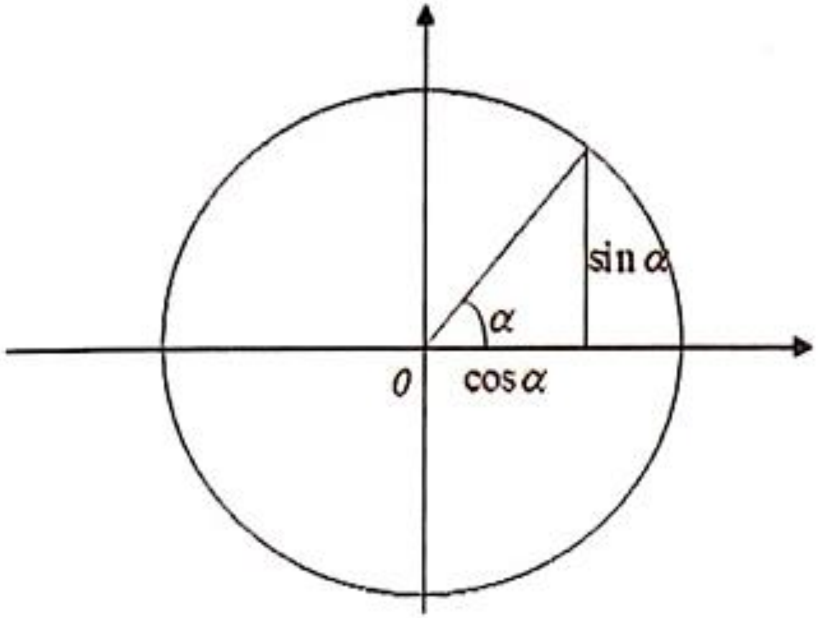
1.1 Trigonometric concepts

Then, $\sin\alpha = \dfrac{opposite\ side}{hypotenuse}$,

$\cos\alpha = \dfrac{adjacent\ side}{hypotenuse}$, $\tan\alpha = \dfrac{opposite}{adjascent}$

$\csc\alpha = \dfrac{hypotenuse}{opposite} = \dfrac{1}{\sin\alpha}$, $\sec\alpha = \dfrac{hypotenuse}{adjascent} = \dfrac{1}{\cos\alpha}$ and

$\cot\alpha = \dfrac{adjacent}{opposite} = \dfrac{1}{\tan\alpha}$



If the circle is unit, the radius is 1 and hence

$$\cos\alpha = adjacent\ side = \frac{x-coordinate}{1} \Rightarrow |\cos\alpha| \le 1$$

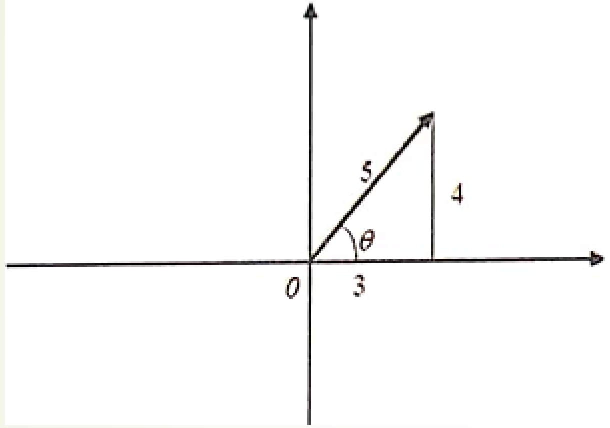
$$\sin\alpha = opposite\ side = \frac{y-coordinate}{1} \Rightarrow |\sin\alpha| \le 1$$

$$\tan\alpha = \frac{opposite\ side}{adjacent\ side} = \frac{y-coordinate}{x-coordinate}$$

1.1 Trigonometric concepts

- **Examples 10:**

Calculate the six trigonometric values for the diagram.



- **Solution:**

Use $adjascent = 3, opposite = 4\ and\ hypotenuse = 5$

$$\sin\theta = \frac{opposite}{hypotenuse} = \frac{4}{5} \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{5}{4}$$

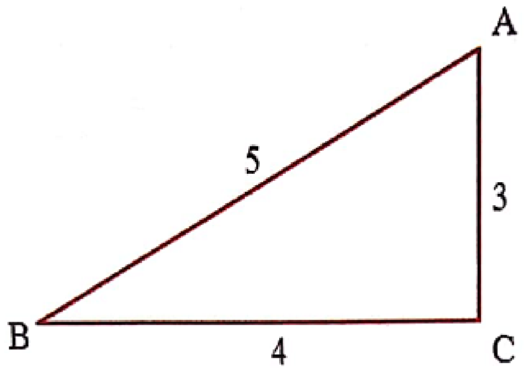
$$\cos\theta = \frac{adjascent}{hypotenuse} = \frac{3}{5} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{5}{3}$$

$$\tan\theta = \frac{opposite}{adjascent} = \frac{4}{3} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{3}{4}$$

1.1 Trigonometric concepts

- **Examples 11:**

For each angle, calculate the reciprocal trigonometric values



- **Solution:**

Angle	*cosecant* $= \frac{1}{sine} = \frac{hypotenuse}{opposite}$	*secant* $= \frac{1}{cosine} = \frac{hypothenuse}{adjacent}$	*cotangent* $= \frac{1}{tangent}$
A	$\frac{5}{4}$	$\frac{5}{3}$	$\frac{3}{4}$
B	$\frac{5}{3}$	$\frac{5}{4}$	$\frac{4}{3}$
$C = 90^0$	$\frac{5}{5} = 1$	$\frac{5}{0}$ *which does not exist*	$\frac{0}{5} = 0$

1.1 Trigonometric concepts

- **Examples 12:**

A positive angle, θ, is in the second quadrant. If $\cos\theta = -\frac{3}{4}$ find the values of the other primary trigonometric values.

- **Solution:**

Let h, x and y be hypotenuse, adjacent and opposite side respectively.

$$\cos\theta = -\frac{3}{4} \Leftrightarrow \frac{x}{h} = -\frac{3}{4}.$$

Since $h > 0$, thus $h = 4$ and $x = -3$.

$$h^2 = x^2 + y^2 \Rightarrow 16 = 9 + y^2$$

$$\Leftrightarrow y^2 = 7 \Rightarrow y = \pm\sqrt{7}$$

As θ is in the second quadrant, $y > 0 \Rightarrow y = \sqrt{7}$.

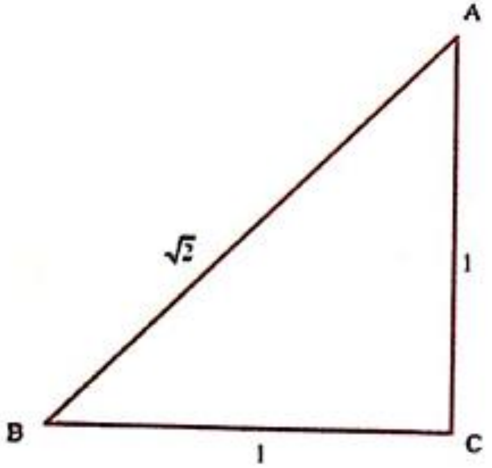
Hence the other primary trigonometric values are

$$\sin\theta = \frac{y}{h} = \frac{\sqrt{7}}{4} \text{ and } \tan\theta = \frac{x}{y} = -\frac{\sqrt{7}}{3}.$$

1.1 Trigonometric concepts

Trigonometric number of special angles 30^0, 45^0, 60^0

As these angles are often used, it is better to keep in your mind their trigonometric ratios in fraction form.



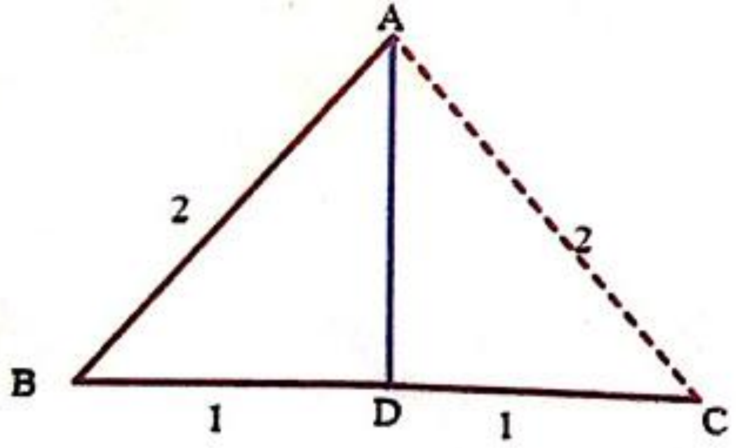
In the figure on the *left*, *ABC* is an isosceles right angled triangle with $BC = CA = 1$. Hence $AB = \sqrt{2}$ and $\angle A = \angle B = 45^\circ$.

Then

$$\sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \cos 45^\circ$$

$$\tan 45^\circ = 1$$

1.1 Trigonometric concepts



In the figure on the *left*, ABC is an equilateral triangle with *side* 2. AD is perpendicular bisector of BC, which implies $BD = 1$ and $AD = \sqrt{3}$. $\angle B = 60^\circ$ and $\angle BAD = 30^\circ$.

Then

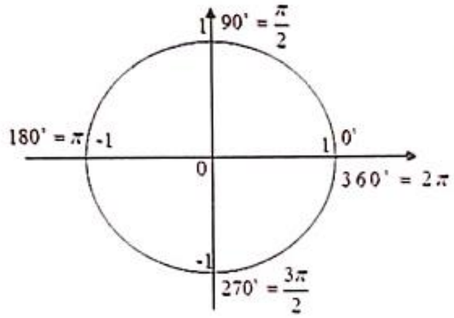
$$\sin 60^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\sin 30^\circ = \cos 60^\circ = \frac{1}{2}$$

$$\tan 60^\circ = \sqrt{3}, \tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Other remarkable angles

Consider the following unit trigonometric circle



From the figure we have

Angle	0^0	90^0	180^0	270^0	360^0
Sin	0	1	0	-1	0
Cos	1	0	-1	0	1

1.1 Trigonometric concepts

- Trigonometric identities

Basic rules

From activity 6 $\cos^2\theta+\sin^2\theta=1$ true for any value of θ.

This relation is called the fundamental formula of trigonometry and is the most frequently used identity in trigonometry.

Dividing this identity by $\cos^2\theta$ *and* $\sin^2\theta$ gives

$$\boxed{\begin{array}{l}1+\tan^2\theta=\sec^2\theta\\ 1+\cot^2\theta=\csc^2\theta\end{array}}$$

- Examples 13:

Simplify $\dfrac{\csc x}{\sec x}$

Solution

$$\frac{\csc x}{\sec x}=\frac{\frac{1}{\sin x}}{\frac{1}{\cos x}}=\frac{\cos x}{\sin x}=\cot x$$

1.1 Trigonometric concepts

- **Examples 14:**

Simplify $\left(\frac{1}{\tan x}+\frac{1}{\cot x}\right)\sin x\cos x$

Solution

$$\left(\frac{1}{\tan x}+\frac{1}{\cot x}\right)\sin x\cos x$$

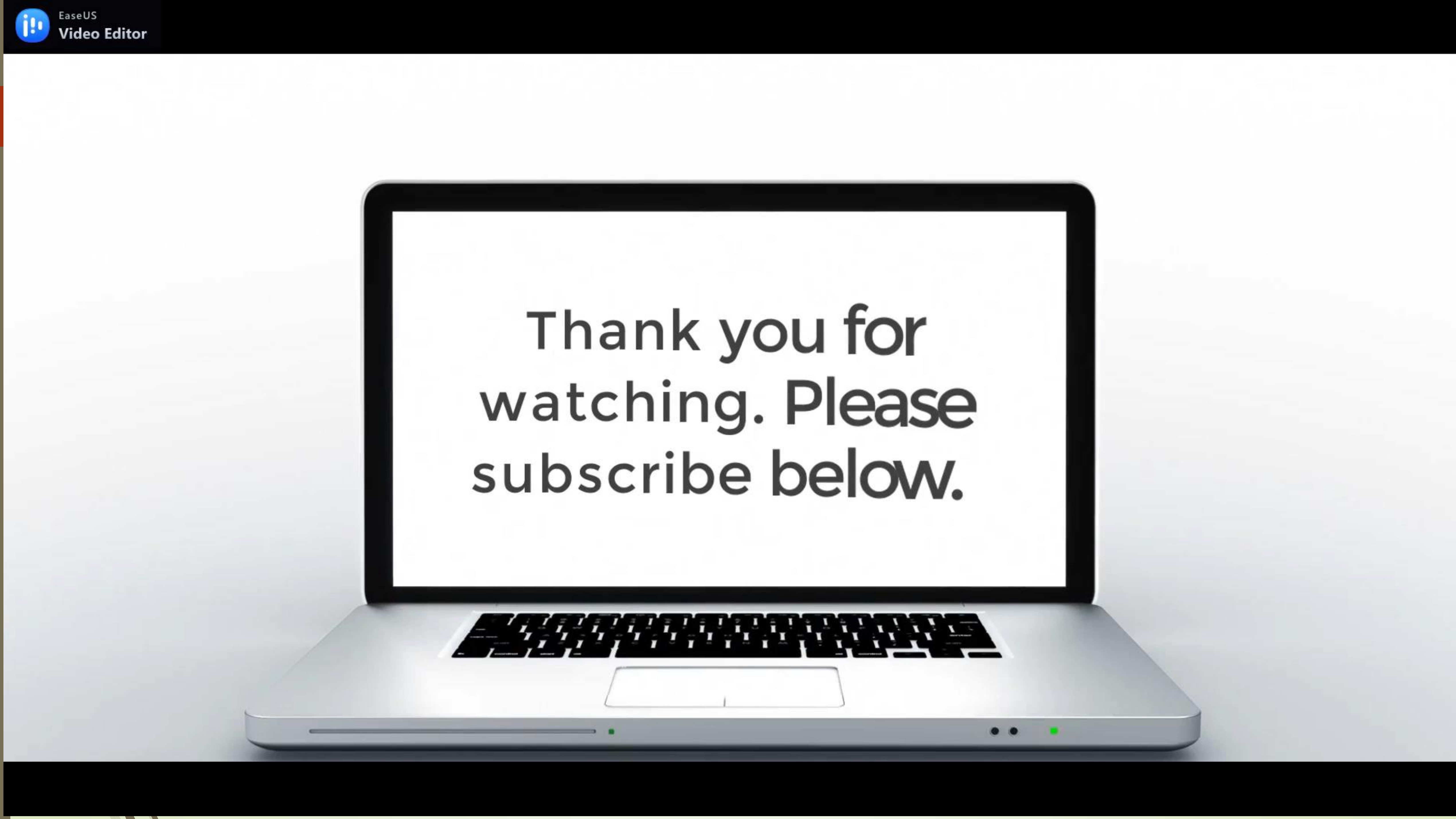
$$=\left(\frac{\cos x}{\sin x}+\frac{\sin x}{\cos x}\right)\sin x\cos x$$

$$=\left(\frac{\cos x\cos x+\sin x\sin x}{\sin x\cos x}\right)\sin x\cos x$$

$$=\cos x\cos x+\sin x\sin x$$

$$=\cos^2 x+\sin^2 x$$

$$=1$$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