THINKBIG O.S. Ltd
www.thinkbig.rw
"In Our Own Way It's Possible"

THINKBIG ONLINE EDUCATION

For more Lessons, visit us on:

- YouTube channel: THINKBIG TUTOR
- Website: www.thinkbig.rw /www.iga.rw

S5, Mathematics , Unit 1: Trigonometric Formulae, Equations and Inequalities

LESSON 1: Trigonometric formulae

Presented by Tutor: **Louis Thelesphore MUNYEMANA**

Unit 1:Trigonometric Formulae, Equations and Inequalities
1.1. Trigonometric formulae

- **1.1.1. Addition and subtraction formulae**
- the addition and subtraction formulae are:

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$
$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$
$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$
$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$

Also:

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$
$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$
$$\cot(x+y) = \frac{\cot x \cot y - 1}{\cot y + \cot x}$$
$$\cot(x-y) = \frac{\cot x \cot y + 1}{\cot y - \cot x}$$

1.1. Trigonometric formulae

- Addition and subtraction formulae are useful when finding **trigonometric number** of some angles.
- **Example 1.1**

Use addition and subtraction formulae to find $\cos 75^0$

- **Solution**

$$\cos 75^\circ = \cos\left(45^\circ + 30^\circ\right)$$

$$= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$$

$$= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\frac{1}{2}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

1.1. Trigonometric formulae

- **Example 1.2**

Use addition and subtraction formulae to find $\sin\frac{\pi}{12}$

- **Solution**

$$\sin\frac{\pi}{12} = \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{6} - \cos\frac{\pi}{4}\sin\frac{\pi}{6}$$

$$= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

1.1. Trigonometric formulae

- **Example 1.3**

Use addition and subtraction formulae to find $\tan\frac{5\pi}{3}$

- **Solution**

$$\tan\frac{5\pi}{3} = \tan\left(2\pi - \frac{\pi}{3}\right)$$

$$= \frac{\tan 2\pi - \tan\frac{\pi}{3}}{1 + \tan 2\pi \tan\frac{\pi}{3}}$$

$$= \frac{0 - \sqrt{3}}{1 + 0} = -\sqrt{3}$$

1.1. Trigonometric formulae
1.1.2. Double angle formulae

$$\cos^2 x + \sin^2 x = 1$$

- This relation is called the fundamental relation of trigonometry.

From this relation, we can write

$$\cos^2 x = 1 - \sin^2 x \text{ and } \sin^2 x = 1 - \cos^2 x$$

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

$$\cot 2x = \frac{\cot^2 x - 1}{2\cot x}$$

$\cos 2x = 1 - 2\cos^2 x$

1.1. Trigonometric formulae
1.1.2. Double angle formulae

- **Example 1.4**

Express cos 4x in function of sin x only

- **Solution.**

$$\cos 4x = \cos 2(2x) = 1 - 2\sin^2 2x$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$= 1 - 2(2\sin x \cos x)^2$$

$$= 1 - 2(4\sin^2 x \cos^2 x)$$

$$= 1 - 8\sin^2 x \cos^2 x$$

$$= 1 - 8\sin^2 x(1 - \sin^2 x)$$

$$= 1 - 8\sin^2 x + 8\sin^4 x$$

1.1. Trigonometric formulae
1.1.2. Double angle formulae

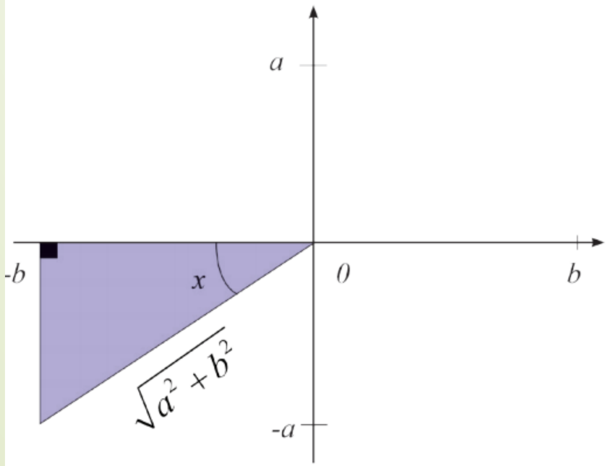
- **Example 1.5**

Given that $\tan x = \dfrac{a}{b}$ and $\pi \le x \le \dfrac{3\pi}{2}$, evaluate

a) $\sin 2x$ b) $\tan 2x$

- **Solution.**

The given information produces the triangle shown below. Note the signs associated with a and b. The Pythagorean Theorem is used to find the hypotenuse.



Hint:

$$\sin x = \frac{opposite\ side}{hypothenuse}$$

$$\cos x = \frac{adjacent\ side}{hypothenuse}$$

$$\tan x = \frac{opposite\ side}{adjacent\ \ side}$$

1.1. Trigonometric formulae
1.1.2. Double angle formulae

a) $$\sin 2x = 2\sin x\cos x$$
$$= 2\cdot\frac{-a}{\sqrt{a^2+b^2}}\cdot\frac{-b}{\sqrt{a^2+b^2}}$$
$$= \frac{2ab}{a^2+b^2}$$

b) $$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$
$$= \frac{2\frac{a}{b}}{1-\left(\frac{a}{b}\right)^2}$$
$$= \frac{2ab}{b^2-a^2}$$

1.1. Trigonometric formulae
1.1.3. Half angle formulae

- The half angle formulae are:

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$\cos\frac{x}{2} = \pm\sqrt{\frac{\cos x+1}{2}}$$

$$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x} \quad \text{or} \quad \tan\frac{x}{2} = \frac{\sin x}{1+\cos x}$$

1.1. Trigonometric formulae
1.1.3. Half angle formulae

- **Example 1.6**

Using the half angle formula, find the exact value of $\cos 15^0$.

- **Solution**

15^0 is in first quadrant, then $\cos 15^0$ must be positive.

$$\cos 15^0 = \cos\left(\frac{1}{2}\left(30^0\right)\right)$$

$$= \sqrt{\frac{1+\cos 30^0}{2}} \qquad = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}$$

$$= \frac{\sqrt{2+\sqrt{3}}}{2}$$

1.1. Trigonometric formulae
1.1.4. Transformation of product in sum

- The formulae for transforming product in sum are:

$$\cos x \cos y = \frac{1}{2}\left[\cos(x+y) + \cos(x-y)\right]$$

$$\sin x \sin y = -\frac{1}{2}\left[\cos(x+y) - \cos(x-y)\right]$$

$$\sin x \cos y = \frac{1}{2}\left[\sin(x+y) + \sin(x-y)\right]$$

$$\cos x \sin y = \frac{1}{2}\left[\sin(x+y) - \sin(x-y)\right]$$

1.1. Trigonometric formulae
1.1.4. Transformation of product in sum

- **Example 1.7**

Transform in sum the product ***sin3x cos 4 x.***

- **Solution**

$$\sin 3x \cos 4x = \frac{1}{2}\left[\sin\left(3x+4x\right)+\sin\left(3x-4x\right)\right]$$

$$= \frac{1}{2}\left[\sin 7x+\sin\left(-x\right)\right]$$

$$= \frac{1}{2}\left[\sin 7x-\sin x\right]$$

1.1. Trigonometric formulae
1.1.5. Transformation of sum in product

- he formulae for transforming sum in product are:

$$\cos p + \cos q = 2\cos\frac{p+q}{2}\cos\frac{p-q}{2}$$

$$\cos p - \cos q = -2\sin\frac{p+q}{2}\sin\frac{p-q}{2}$$

$$\sin p + \sin q = 2\sin\frac{p+q}{2}\cos\frac{p-q}{2}$$

$$\sin p - \sin q = 2\cos\frac{p+q}{2}\sin\frac{p-q}{2}$$

1.1. Trigonometric formulae
1.1.5. Transformation of sum in product

- **Example 1.8**

Transform in product the sum ***sin3x+sin 4x***

- **Solution**

$$\sin 3x + \sin 4x = 2\sin\frac{3x+4x}{2}\cos\frac{3x-4x}{2}$$

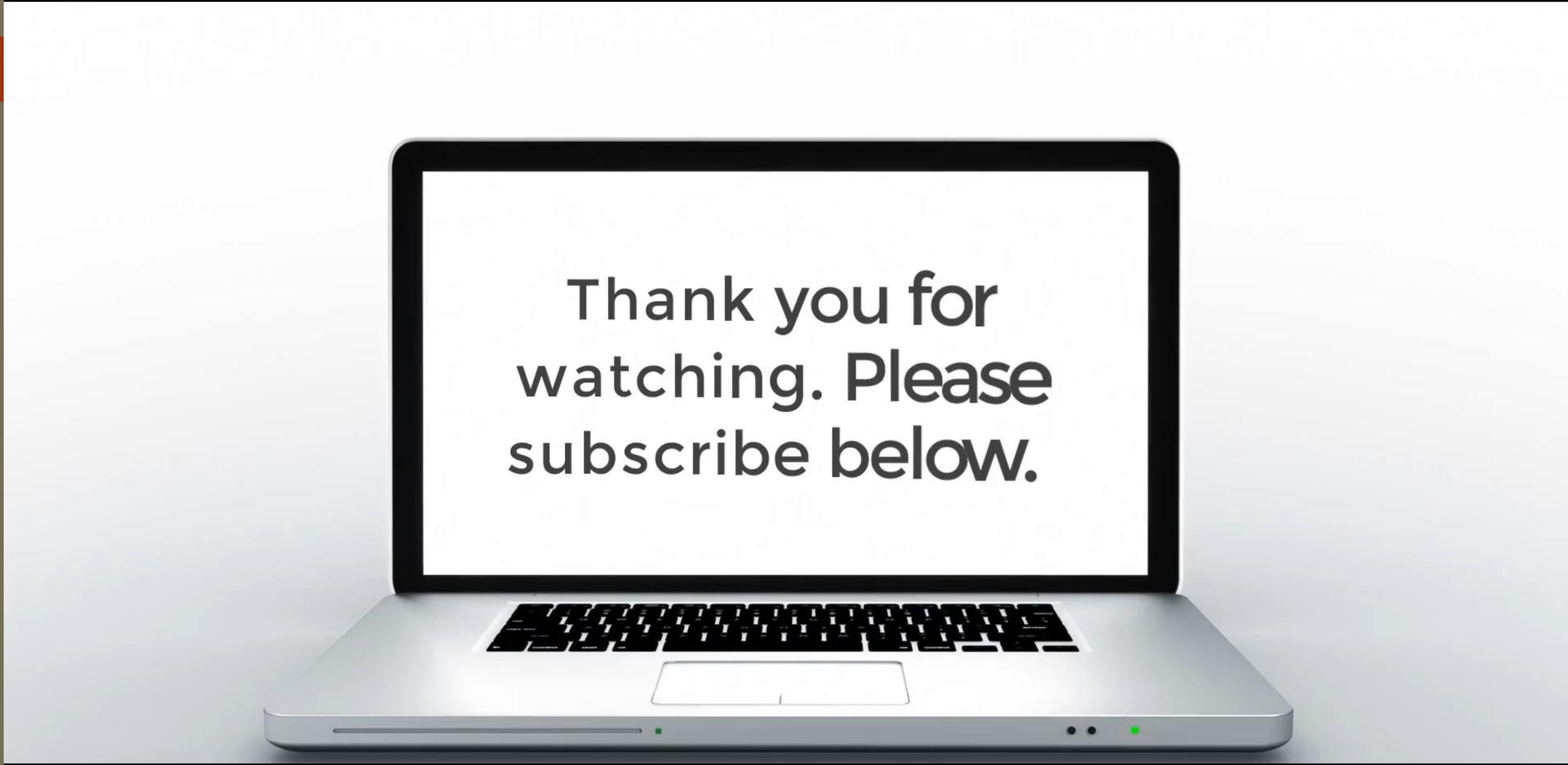
$$= 2\sin\frac{7x}{2}\cos\frac{x}{2}$$

THINKBIG ONLINE EDUCATION

MUSHOBORA NO KUDUKURIKIRA KUMBUGA ZIKURIKIRA AHO MUSANGA AMASOMO ANYURANYE UBU TURI KWIBANDA KU MASOMO YA SCIENCE N'IMIBARE(MATHS, SET, PHYSICS, BIOLOGY AND CHEMISTRY) YO KUVA P6 –S6, MUDUSURE MUKORE NA SUBSCRIBE TUBANE IGIHE CYOSE:

- YouTube channel: THINKBIG TUTOR
- Website: www.thinkbig.rw /www.iga.rw

"On your own way it's possible, don't give up!!!!!"


Thank you for watching. Please subscribe below.