MATHEMATICS II

029

12/11/2019 8:30 AM-11:30 AM

Rwanda Education Board

ADVANCED LEVEL NATIONAL EXAMINATION, 2019

SUBJECT: MATHEMATICS

COMBINATIONS:

- **MATHEMATICS-CHEMISTRY-BIOLOGY (MCB)**
- **MATHEMATICS-COMPUTER SCIENCE-ECONOMICS (MCE)**
- **MATHEMATICS-ECONOMICS-GEOGRAPHY (MEG)**
- **MATHEMATICS-PHYSICS-COMPUTER SCIENCE (MPC)**
- **MATHEMATICS-PHYSICS-GEOGRAPHY (MPG)**
- **PHYSICS-CHEMISTRY-MATHEMATICS (PCM)**

DURATION: 3 HOURS

INSTRUCTIONS:

1) Write your names and index number on the answer booklet as written on your registration form and **DO NOT** write your names and index number on additional answer sheets of paper if provided.
2) Do not open this question paper until you are told to do so.
3) This paper consists of **two** sections **A** and **B**.

   SECTION A: Attempt **all** questions. **(55Marks)**

   SECTIONB: Attempt **only three** questions. **(45Marks)**
4) **Geometrical instruments and silent non-programmable calculators May be used.**
5) Use only a **blue** or **black** pen

SECTION A: ATTEMPT ALL QUESTIONS (55Marks)

1) Show that $\frac{1}{2}(\cos 2x - \sin 2x) = \cos^2 x - \cos x \sin x - \frac{1}{2}$ **(3marks)**

2) Solve in $\mathbb{R}$ $4e^{1+3x} - 9e^{5-2x} = 0$ **(3marks)**

3) Assume that $x = 4\sin(2y+6)$; find $\frac{dy}{dx}$ in terms of x **(3marks)**

4) Let $f:\mathbb{R}\to\mathbb{R}$ be a function defined by $f(x) = -2x+8$

(i) Find $f^{-1}(x)$ **(1mark)**

(ii) From (i) prove that $f^{-1}(x)$ is an inverse function of $f(x)$ **(3marks)**

5) (i) Find the general solution of the differential equation $\frac{d^2y}{dx^2} = x + \sin x$ **(2marks)**

(ii)Find the particular solution of the equation given in (i) respecting the following initial conditions : $y = 0$ and $\frac{dy}{dx} = -1$ when $x = 0$ **(3marks)**

6) Calculate the limit : $\lim_{x\to\infty}\left(\frac{x}{x+1}\right)^{x+2}$ **(3marks)**

7) Consider the conics given by $x^2+2x+y^2-8y+8=0$

a) Write the standard form of the circle. **(2marks)**

b) Find the centre. **(1mark)**

c) Find its radius. **(1mark)**

8) a) Transform in product $\sin 4x+\sin 5x$ **(1mark)**

b) Transform in sum $\sin 4x+\cos 5x$ **(2marks)**

9) If the position of an object after t hours is given by $f(t)=\frac{t}{t+1}$

a) Is this object moving to the right or left at $t=10$ hours. Justify your answer. **(2marks)**

b) Does the object ever stop moving? Justify your answer. **(2marks)**

10) a)The release of chlorofluorocarbons used in air conditioners and household spray (hair spray , shaving cream,...) destroys the ozone in the upper atmosphere. The quantity of ozone Q, is decaying exponentially at a continuous rate of 0.25% per year. How long will it take for a half of ozone to disappear? **(2marks)**
(Express the answer in years)

Assume that the quantity of ozone is modeled by $Q=Q_0e^{-kt}$, where Q_0 is the initial quantity of an ozone, k is a continuous rate of decaying and t is time in years.

b) (i) Show that $p\Rightarrow q$ and $\sim p\vee q$ are logically equivalent. **(2marks)**
Justify your answer

(ii) How do we call this tautology? **(1mark)**

11) Calculate the integral $\int_0^{\frac{\pi}{2}} \sin x \sin 2x dx$ **(3marks)**

12) The marginal revenue function of a commodity is given as $MR = 12 - 3x^2 + 4x$

Find:

a) The total revenue function of the commodity. **(2marks)**

b) The demand function of the commodity. **(2marks)**

Where $R(x)$ is the total revenue function; MR is the marginal revenue function

With $MR = \frac{d}{dx}[R(x)]$ and p is the demand function such that $p = \frac{R(x)}{x}$.

13) a) If the line L which passes through the point $P = (1,2,3)$ and parallel to the vector $\vec{v} = \vec{i} - 2\vec{j} + 3\vec{k}$. Find its position vector **(1mark)**

b) Given that $\vec{u} = -2\vec{i} + \vec{j} + 2\vec{k}$ and $\vec{v} = -3\vec{i} - 2\vec{j} + 6\vec{k}$ are vectors equations of two straight lines in space. Determine the angle between the two vectors. **(2marks)**

14) a) If C and D are the points with affixes $Z_C = 2 - i$ and $Z_D = 5 + 2i$

Find $\overline{CD}$ **(2marks)**

b) Linearize $\cos^2 x \sin x$ **(2marks)**

15) An airplane flying horizontally 1000m above the ground is observed at an elevation of 60^0. If after 10 seconds its elevation is observed to be 30^0, find the uniform speed per hour of the airplane. **(3marks)**

SECTIONB: ATTEMPT 3 QUESTIONS (45MARKS)

16) a) If the ellipse is given by the equation : $9x^2 + 49y^2 = 441$

(i) Write the standard form of the ellipse **(2marks)**

(ii) Determine its centre. **(1mark)**

(iii) Determine its foci. **(2marks)**

(iv) Determine its vertices. **(1mark)**

b) The gradient of the curve C is given by $\frac{dy}{dx} = (3x-1)^2$

The point $P(1,4)$ lies on C.

(i) Find an equation of normal to C at P. **(2marks)**

(ii) Find an equation for the curve C in the form $y = f(x)$ **(4marks)**

c) Find the length of a line whose slope is -3 given that line extends from $x = 1$ to $x = 6$ **(3marks)**

17) a) Evaluate the following integral $\int \sin^6 x \cos^3 x dx$ **(4marks)**

b) Find by Maclaurin series of $\int \ln(1-p)dp$. **(3marks)**

c) A heated metal ball is dropped into a liquid. As the ball cools, its temperature T^0C , t minutes after it enters the liquid, is given by $T = 400e^{-005t} + 25$, $t \geq 0$

(i) Find the temperature of the ball as it enters the liquid. **(1mark)**

(ii) Find the value of t for which $T = 300^0 C$, giving your answer to 2 decimal Places. **(4marks)**

(iii) Find the rate at which the temperature of the ball is decreasing at the instant when $t = 50$. Give your answer in ^0C per minute to 2 decimal places. **(3marks)**

18) a) If 10,000 FRW is deposited in an account paying a compound interest rate of 5% per year continuously, how long does it take for the balance in the account to reach 15,000FRW ? **(4marks)**

b) i) Write down the Cayley table for addition modulo 5 on the set $\mathbb{Z}$ or $(\mathbb{Z}_5,+) = ((\text{mod } 5),+)$ **(2marks)**

ii) Verify if $(\mathbb{Z}_5,+)$ Cayley table found in b) i) above is a commutative group. **(3marks)**

c) An object starts from rest and has an acceleration of $a(t) = t^3$. What is its;

(i) Velocity after 6 seconds? **(3marks)**

(ii) Position after 6 seconds? **(3marks)**

19) The scores of 6 students in their Chemistry and Biology subject tests are:

Chemistry (x)	3	6	5	8	11	9
Biology (y)	2	3	4	6	5	8

a) Complete the table below: **(8marks)**

x	y	$x-\bar{x}$	$y-\bar{y}$	$(x-\bar{x})^2$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
$\sum_{i=1}^{6} x_i$	$\sum_{i=1}^{6} y_i$			$\sum_{i=1}^{6}(x_i-\bar{x})^2$	$\sum_{i=1}^{6}(y_i-\bar{y})^2$	$\sum_{i=1}^{6}(x_i-\bar{x})(y_i-\bar{y})$
$\bar{x}$	$\bar{y}$					

b) Find the standard deviation for Chemistry scores. **(1mark)**

c) Find the standard deviation for Biology scores. **(1mark)**

d) Find the covariance $\text{cov}(x,y)$ of two subject scores. **(2marks)**

e) Find the coefficient of correlation between Chemistry and Biology scores. **(2marks)**

f) Interpret the coefficient of correlation found in (v) above. **(1mark)**

20) a) Solve the equation $1+i=z^2$ **(5marks)**

b) A particular species of orchid is being studied. The population p and time t years after the study started is assumed to be $p=\dfrac{2800ae^{0.2t}}{1+ae^{0.2t}}$ where a is a constant.

Given that there were 300 orchids when the study started,

(i) Show that $a=0.12$ **(3marks)**

(ii) Use the equation with $a = 0.12$ to predict the number of years before The population of orchids reaches 1850. **(4marks)**

(iii) Show that $p = \frac{336}{0.12 + e^{-0.2t}}$ **(1mark)**

(iv) Hence show that the population cannot exceed 2800. **(2marks)**

MARKING SCHEME MATHS 2019

MARKING GUIDES OF MATHEMATICS NATIONAL EXAMINATION

1. $\frac{1}{2}(\cos 2x - \sin 2x) = \frac{1}{2}(\cos^2 x - \sin^2 x - 2\sin x \cos x)$

$$= \frac{1}{2}(\cos^2 x - (1 - \cos^2 x) - 2\sin x \cos x)$$
$$= \frac{1}{2}(2\cos^2 x - 1 - 2\sin x \cos x)$$
$$= \frac{1}{2}(\cos^2 x - 1 + \cos^2 x - 2\sin x \cos x)$$
$$= \cos^2 x - \frac{1}{2} - \sin x \cos x$$
$$= \cos^2 x - \sin x \cos x - \frac{1}{2}$$

2. $4e^{1+3x} - 9e^{5-2x} = 0$

$4e^{1+3x} = 9e^{5-2x}$

$\frac{e^{1+3x}}{e^{5-2x}} = \frac{9}{4}$

$e^{1+3x-5+2x} = \frac{9}{4}$

$e^{-4+5x} = \frac{9}{4}$

$\ln e^{-4+5x} = \ln \frac{9}{4}$

$-4 + 5x = \ln \frac{9}{4} \Rightarrow 5x = \ln \frac{9}{4} + 4 \Rightarrow x = \frac{1}{5}(n\frac{9}{4} + 4) \Rightarrow x = 0.961286$

3. $x = 4\sin(2y + 6)$

$$1=8\cos(2y+6)\frac{dy}{dx}$$

$$\frac{dy}{dx}=\frac{1}{8\cos(2y+6)}\Rightarrow let\frac{x}{4}=\sin(2y+6)\Rightarrow 2y+6=\arcsin\frac{x}{4}$$

$$\frac{dy}{dx}=\frac{1}{8\cos\left(\arcsin\frac{x}{4}\right)}=\frac{1}{8\sqrt{\frac{16-x^2}{16}}}=\frac{1}{2\sqrt{16-x^2}}$$

4. Let $f(x)=-2x+8$

i) So that y is image of x under f

$$y=-2x+8$$
$$2x=8-y$$
$$x=\frac{8-y}{2}$$

To find $f^{-1}(x)$ replace y by x then we get $f^{-1}(x)=\frac{8-x}{2}$

ii) Verify that $f(f^{-1}(x))=f^{-1}(f(x))=x$

$$f(f^{-1}(x))=f(\frac{8-x}{2})=-2(\frac{8-x}{2})+8=-8+x+8=x$$

Find $f^{-1}(f(x))=f^{-1}(-2x+8)=\frac{8-(-2x+8)}{2}=\frac{2x}{2}=x$

Hence $f(f^{-1}(x))=f^{-1}(f(x))=x$

5. i) $\frac{d^2y}{dx^2}=x+\sin x$

$\frac{d}{dx}\left(\frac{dy}{dx}\right)=x+\sin x$ by integrating

$$\int d\left(\frac{dy}{dx}\right)=\int(x+\sin x)dx$$

$$\frac{dy}{dx}=\int(x+\sin x)dx=\frac{x^2}{2}-\cos x+c_1$$

$$\int dy=\int\left(\frac{x^2}{2}-\cos x+c_1\right)dx\Rightarrow y=\frac{x^3}{6}-\sin x+xc_1+c_2$$

ii) use $\frac{dy}{dx}=-1$ and x=0

we get $-1=\frac{0}{2}-\cos 0+c1$

$$-1=-1+c1 \Rightarrow c1=0$$

Use y=0 and x=0, we get

$$0=\frac{0}{6}-\sin 0+0+c2$$
$$0=0-0+0+c2 \Rightarrow c2=0$$

Therefore the particular solution is $y_p=\frac{x^3}{6}-\sin x$

6. $\lim_{x\to+\infty}\left(\frac{x}{x+1}\right)^{x+2}$

Use the formula $\lim_{x\to\pm\infty}[f(x)]^{g(x)}=e^{\lim_{x\to\pm\infty}[f(x)-1]g(x)}$

$$\lim_{x\to+\infty}\left(\frac{x}{x+1}\right)^{x+2}=1(IF)$$

$$\lim_{x\to+\infty}\left(\frac{x}{x+1}\right)^{x+2}=e^{\lim_{x\to+\infty}\left(\frac{x}{x+1}-1\right)(x+2)}=e^{\lim_{x\to+\infty}\left(\frac{x-x-1}{x+1}\right)(x+2)}=e^{\lim_{x\to+\infty}\left(\frac{-x-2}{x+1}\right)}=e^{-1}=\frac{1}{e}$$

7. $x^2+2x+y^2-8y+8=0$

$(x^2+2x)+(y^2-8y)+8=0 \Rightarrow (x+1)^2-1+(y-4)^2-16+8=0$

$(x+1)^2+(y-4)^2-9=0 \Rightarrow (x+1)^2+(y-4)^2=9$

Central is C(-1,4)

Radius R=3

8. a) $\sin 4x+\sin 5x=2\sin\left(\frac{4x+5x}{2}\right)\cos\left(\frac{4x-9x}{2}\right)=2\sin\frac{9x}{2}\cos\frac{x}{2}$

b) sin4x+sin5x: is already transformed in sum

-impossible

9. a) $f(t)=\frac{t}{t+1} \Rightarrow f'(t)=\frac{t}{(t+1)^2}$ then

$$f'(0)=\frac{t}{(10+1)^2}=\frac{1}{121}$$

Therefore, the velocity at t=10 is possible then the object is moving to the right

b) the object will stop moving iff f'(t)=0, for all values t positive, since

$f'(t) \neq 0$, the object will never stop moving.

10. As Q_0 Is the initial quantity of ozone and t is in year $Q = Q_0 e^{-0.0025t}$ we need to find the value of t

$$Q = \frac{Q_0}{2}$$

$$\frac{Q_0}{2} = Q_0 e^{-0.0025t} \Rightarrow \frac{1}{2} = e^{-0.0025t} \Rightarrow \ln\frac{1}{2} = \ln e^{-0.0025t} \Rightarrow \ln\frac{1}{2} = -0.0025t$$

$$t = \frac{\ln(\frac{1}{2})}{-0.0025} = 277.25 \approx 277 \, years$$

Then a half present atmosphere ozone will be disappearing in n277 years

b)

p	q	$p \Rightarrow q$	$\neg p$	$\neg p \cup q$	$(p \Rightarrow q) \Leftrightarrow (\neg p \cup q)$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

At the last column show us that two statements are identic. The two statement are logically equivalent. This tautology is called conditional disjunction.

11. $I = \int_0^{\frac{\pi}{2}} 2\sin x \sin 2x dx$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} 2\sin 2x \sin x dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} (\cos x - \cos 3x) dx = \frac{1}{2}\int_0^{\frac{\pi}{2}} \cos x dx - \frac{1}{2}\int_0^{\frac{\pi}{2}} \cos 3x) dx$$

$$= \frac{1}{2}[\sin x]_0^{\frac{\pi}{2}} - \frac{1}{2}\left[\frac{\sin 3x}{3}\right]_0^{\frac{\pi}{2}} = \frac{1}{2}\left[\sin\frac{\pi}{2} - \sin 0\right] - \frac{1}{6}\left[\frac{\sin 3\frac{\pi}{2}}{3} - \frac{\sin 3.0}{3}\right]$$

$$= \frac{1}{2}[1-0] - \frac{1}{6}[-1-0] = \frac{1}{2} + \frac{1}{6} = \frac{2}{3}$$

12. a) $MR = 12 - 3x^2 + 4x$ and

$$MR = \frac{d(R(x))}{dx} \Rightarrow R(x) = \int (MR)dx = \int (12 - 3x^2 + 4x)dx = 12x - x^3 + 2x^2 + k$$

$$MR = 12x - x^3 + 2x^2 + k$$

b) $P = \frac{R(x)}{x} = \frac{12x - x^3 + 2x^2 + k}{x} = 12 - x^2 + 2x + \frac{k}{x}$

13. a) P(1,2,3), $\vec{v} = -2\vec{j} + \vec{i} + 3\vec{k}$

position vector is $\vec{p} = \vec{i} + 2\vec{j} + 3\vec{k}$ or $\overrightarrow{op} = (1,2,3)$

the line does not have a position vector

b) if θ is the angle between two vectors

$$\vec{u} = -2\vec{i} + \vec{j} + 2\vec{k}$$
$$\vec{v} = -3\vec{j} - 2\vec{j} + 6\vec{k}$$
$$\cos\theta = \frac{\vec{u}\vec{v}}{\|\vec{u}\|\|\vec{v}\|} = \frac{6-2+12}{\sqrt{9}\sqrt{49}} = \frac{16}{21}$$
$$\theta = \cos^{-1}\left(\frac{16}{21}\right) = 40.37^0$$

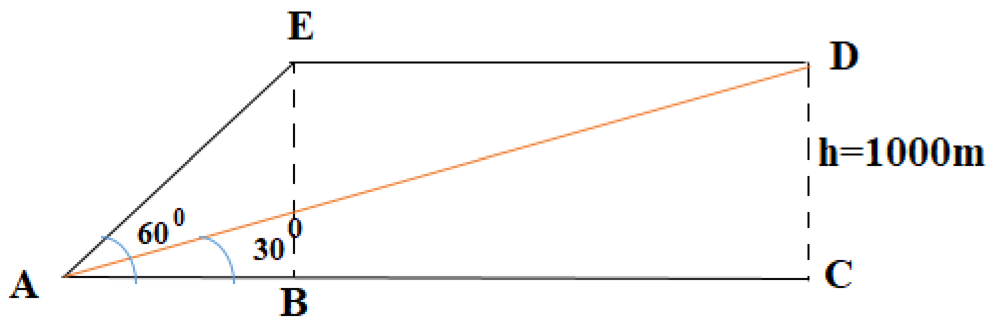
14. a) $z_c = 2 - i$ and $z_d = 5 - 2i$

$$|z_d - z_c| = \overline{cd} = |5 + 2i - (2 - i)| = |3 + 3i| = \sqrt{9+9} = 3\sqrt{2}$$

b) $\cos^2 x \sin x = \frac{1}{2}(1+\cos 2x)\sin x = \frac{1}{2}\sin x + \frac{1}{2}\sin x \cos 2x$

$$= \frac{1}{2}\sin x + \frac{1}{4}(\sin 3x + \sin(-x))$$
$$= \frac{1}{4}\sin x + \frac{1}{4}\sin 3x - \frac{1}{4}\sin x$$
$$= \frac{1}{4}\sin xx + \frac{1}{4}\sin 3x$$

15.



$$\tan 30^0 = \frac{CD}{AC} \Rightarrow AC = \frac{CD}{\tan 30^0}$$
$$\tan 60^0 = \frac{BE}{AB} \Rightarrow AB = \frac{BE}{\tan 60^0} = \frac{100}{\sqrt{3}}$$

$$\tan 30^0 = \frac{CD}{AC} \Rightarrow AC = \frac{CD}{\tan 30^0}$$

$$\tan 30^0 = \frac{CD}{AC} \Rightarrow AC = \frac{100}{\frac{1}{\sqrt{3}}} = 100\sqrt{3}$$

The distance travelled by airplane is ED=BC=AC-AB then $ED = 100\sqrt{3} - \frac{100}{\sqrt{3}} = \frac{200}{\sqrt{3}}$

- Uniform speed of the air plane is $Velocity = \frac{distance}{time}$

  $$V = \frac{\frac{200}{\sqrt{3}}}{10} = \frac{2000}{\sqrt{3}} \times \frac{1}{100} = \frac{200}{\sqrt{3}} m/s$$

  Velocity $v = \frac{200}{\sqrt{3}} \times 3600 m/h = 415{,}692 m/h = 415.692 km/h$

SECTION B

16. i)

$$9x^2 + 49y^2 = 441$$

$$\frac{9x^2}{441} + \frac{49y^2}{441} = \frac{441}{441}$$

$$\frac{x^2}{49} + \frac{y^2}{9} = 1$$

Its standard form is

$$\frac{x^2}{49}+\frac{y^2}{9}=1$$

ii)The centre of ellipse is (0,0)

iii)The foci

$$a^2-b^2=c^2$$
$$49-9=c^2$$
$$40=c^2$$
$$c^2=\pm\sqrt{40}$$

foci are $(\pm 2\sqrt{10},0)$

iii) The vertices are (-7,0), (7,0),(0,-3)and(0,3)

b) i) $\frac{dy}{dx}=(3x-1)^2$

Equation of normal is : $N\equiv y-y_0=-\frac{1}{f'(x_0)}(x-x_0)$

$f'(1)=4$

$$N\equiv y-4=\frac{-1}{4}(x-1)$$
$$N\equiv 4y=-x+17$$
$$N\equiv y=-\frac{x}{4}+\frac{17}{4}$$

$$(3x-1)^2=(3x)^2-2.3x.1=1$$
$$9x^2-6x+1$$

ii) $\frac{dy}{dx}=9x^2-6x+1$

$$y=\int(9x^2-6x+1)dx$$
$$y=\frac{9x^3}{3}-\frac{6x^2}{2}+x+c$$

substitute (1,4) to find the value of c: c=3

The equation of curve is $y=3x^3-3x^2+x+3$

c)The length of the line when $f'(x) = -3$ is given by

$$L = \int_1^6 \sqrt{1+(f'(x))^2}\,dx$$
$$L = \int_1^6 \sqrt{10}\,dx$$
$$L = \sqrt{10}\left[x\right]_1^6$$
$$L = 5\sqrt{10}$$

unit of length.

17. a) $\int \sin^6 x \cos^3 x dx = \int \sin^6 x(1-\sin^2 x)\cos x dx$

Suppose that $u=sinx$

$du=cosxdx$

$$\int \sin^6(1-\sin^2 x)\cos x dx = \int u^6(1-u^2)du$$
$$= \int (u^6 - u^8)du$$
$$= \frac{u^7}{7} - \frac{u^9}{9} + c$$
$$= \frac{1}{7}\sin^7 x - \frac{1}{9}\sin^9 x + c$$

b) Maclaurin series of ln(1-p)

ln(1-p) is $\ln(1-p) = -p - \frac{p^2}{2} - \frac{p^3}{3} - \frac{p^4}{4} ---$

$$\int \ln(1-p)dp = \int (-p - \frac{p^2}{2} - \frac{p^3}{3} - \frac{p^4}{4} ---)dp$$
$$= -\frac{p^2}{2} - \frac{p^3}{6} - \frac{p^4}{12} - \frac{p^5}{20} ---- + c$$

C)i) t=0;

$$T = 400e^0 + 25$$
$$T = 400 + 25$$
$$T = 425^0 C$$

$$300 = 400e^{-0.05t} + 25$$
$$275 = 400e^{-0.05t}$$

ii) $-0.05t = \ln(\frac{275}{400})$

$$t = \frac{-1}{0.05}\ln(\frac{275}{400})$$
$$t = 7.49\,\text{min}$$

iii)Rate of change $\frac{dT}{dt} = -0.05 \times 400e^{-0.05t}$

$$= -20e^{-0.05 \times 50}$$
$$= -1.64^0 C$$

18. a) $p = p_0 e^{rt}$ with r=0.05 and $p_0 = 10000\,frw$

$$p = 15000\,frw$$

$$15000 = 10000e^{0.05t}$$
$$1.5 = e^{0.05t}$$
$$\ln 1.5 = \ln e^{0.05t}$$
$$\ln 1.5 = 0.05t$$
$$t = \frac{\ln 1.5}{0.05} = 8.1093$$

It will take about 8.1years

b) i) $Z_5 = \{0,1,2,3,4\}$

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1

3	3	4	0	1	2
4	4	0	1	2	3

ii) to verify if **(Z_5, +)** Is a commutative group, we have to verify the following properties. The operation **(Z_5, +) is:**

1) **Closure**: Each result inside the table is the element of the set **Z_5**
2) **Associative: Eg: 2+(3+4)=2+2=4=(2+3)+4**
3) There exists an identity element 0 Eg: 4+0=0+4=4;3+0=0+3=3
4) There exists an inverse element
   Eg: 1+4=0, 4 is the inverse of 1; 2+3=0, 3 is the inverse of 2, 0+0=0, o is the inverse of itself
5) Commutative: eg: 1+2=2+1=3. It is illustrated by the data inside the table which are symmetric about the diagonal from the top left to the bottom right. Therefore, addition modulo 5 on the set **Z_5 is a commutative group.**

c) S(0)=0, V(0)=0

i)

$$v(t)=\int a(t)dt=\int t^3 dt=\frac{t^4}{4}+c$$

$$v(0)=\frac{0^4}{4}+c \Rightarrow c=0$$

After 6 seconds $v(6)=\frac{6^4}{4}=324$ of velocity

ii) $s(t)=\int v(t)dt=\int \frac{t^4}{4}dt=\frac{t^5}{20}+c$

$$s(0)=\frac{0^5}{20}+c \Rightarrow c=0$$

$$s(t)=\frac{t^5}{20}$$

For t=6sec $s(6)=\frac{6^5}{20}=388.8$

19. A)

x	y	$x-\bar{x}$	$y-\bar{y}$	$(x-\bar{x})^2$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
3	2	-4	-2.6	16	6.76	10.4
6	3	-1	-1.6	1	2.56	1.6
5	4	-2	-0.6	4	0.36	1.2
8	6	1	1.4	1	1.96	1.4
11	5	4	0.4	16	0.16	1.6
9	8	2	3.4	4	11.56	6.8
Sum=42 Mean=7	28 4.6			42	23.36	23

b) Standard deviation SD for chemistry is found by $\sqrt{\delta^2}$ where $\delta^2 = \frac{1}{n}\sum_{i=1}^{n}\left(x-\bar{x}\right)^2$

$\delta^2 = \frac{42}{6} = 7$, $SD = \sqrt{\delta^2} = \sqrt{7} = 2.64$

c) standard deviation SD For biology is formed by $\sqrt{\delta^2}$ where

$\delta^2 = \frac{1}{n}\sum_{i=1}^{n}\left(y-\bar{y}\right)^2 = \frac{23.36}{6} = 3.89$ variance is 3.89 and SD= $SD = \sqrt{\delta^2} = \sqrt{3.809} = 1.97$

d) $\text{cov}(x,y) = \frac{1}{n}\sum_{i=1}^{n}(x-\bar{x})(y-\bar{y}) = \frac{23}{6} = 3.83$

e) $r = \frac{\text{cov}(x,y)}{\delta_x \delta_y} = \frac{3.87}{2.64\times 1.97} = \frac{3.87}{5.20} = 0.74$

f) The correlation coefficient r=0.74 is strong and positive

20.

a) Let $z = x + yi$, then

$z^2 = (x+yi)^2 = x^2 - y^2 + 2xyi = 1 + i$

Equating real and imaginary part to get $x^2 - y^2 = 1$ and $2xy = 1$

$$\begin{cases} x^2 - y^2 = 1 \\ 2xyi = 1 \end{cases} \Rightarrow y = \frac{1}{2x} \Rightarrow x^2 - \left(\frac{1}{2x}\right)^2 = 1 \Rightarrow 4x^4 - 4x^2 - 1 = 0 \Rightarrow x = \pm\sqrt{\frac{1+\sqrt{2}}{2}}$$

For $y = \frac{1}{2x} = \pm\frac{2}{2\sqrt{\frac{1+\sqrt{2}}{2}}} = \pm\frac{1}{\sqrt{2}\sqrt{1+\sqrt{2}}}$

Therefore, the solution are

$$z_1 = \sqrt{\frac{1+\sqrt{2}}{2}} + i\sqrt{\frac{-1+\sqrt{2}}{2}}$$

$$z_2 = -\sqrt{\frac{1+\sqrt{2}}{2}} - i\sqrt{\frac{-1+\sqrt{2}}{2}}$$

b) i) t=0, p=300

$$P=\frac{2800ae^{0.2t}}{1+ae^{0.2t}}\Rightarrow 300=\frac{2800ae^{0.2\times 0}}{1+ae^{0.2\times 0}}=\frac{2800a}{1+a}$$

$$300(1+a)=2800a\Rightarrow 300=2800a-300a\Rightarrow 2500a=300\Rightarrow a=\frac{300}{2500}=0.12$$

ii) P=1850

$$1850=\frac{2800ae^{0.2t}}{1+ae^{0.2t}}\Rightarrow 300=\frac{2800\times 0.12e^{0.2\times t}}{1+0.12e^{0.2\times t}}=\frac{336e^{0.2\times t}}{1+0.12e^{0.2\times t}}$$

$$1850(1+0.12e^{0.2\times t})=336e^{0.2\times t}\Rightarrow 1850=114e^{0.2\times t}$$

$$0.2t=\ln\frac{1850}{114}\Rightarrow t=\frac{1}{0.2}\ln\frac{1850}{114}=13.9\approx 14\,years$$

Therefore, the number of years before the population of orchids reaches 1850 is about 14years

iii) $P=\frac{2800\times 0.12e^{0.2t}}{1+0.12e^{0.2t}}$ divided by $e^{0.2t}$ $P=\frac{336}{e^{-0.2t}+0.12}$

iv) where $t\to\infty$ $P=\frac{336}{0.12}=2850$

Therefore, the maximum value of P=2850