Mathematics II

029

26/07/2022 **8:30 AM-11:30 AM**

NATIONAL EXAMINATION AND SCHOOL INSPECTION AUTHORITY

ADVANCED LEVEL NATIONAL EXAMINATIONS, 2021-2022

SUBJECT: MATHEMATICS II

COMBINATIONS:

- MATHEMATICS-CHEMISTRY-BIOLOGY **(MCB)**
- MATHEMATICS -COMPUTER SCIENCE-ECONOMICS **(MCE)**
- MATHEMATICS-ECONOMICS-GEOGRAPHY **(MEG)**
- MATHEMATICS -PHYSICS-COMPUTER SCIENCE **(MPC)**
- MATHEMATICS-PHYSICS-GEOGRAPHY **(MPG)**
- PHYSICS-CHEMISTRY-MATHEMATICS **(PCM)**

DURATION: 3 HOURS

INSTRUCTIONS:

1) Write your names and index number on the answer booklet as written on your registration form, and **DO NOT** write your names and index number on additional answer sheets if provided.
2) Do not open this question paper until you are told to do so.
3) This paper consists of **two** sections: **A** and **B**.

   Section A: Attempt **ALL** questions. **(55 marks)**

   Section B: Attempt any **THREE** questions. **(45 marks)**
4) **Geometrical instruments and silent non-programmable calculators may be used.**
5) Use only a **blue** or **black** pen.

SECTION A: ATTEMPT ALL QUESTIONS (55 marks)

1) Evaluate the $\lim_{x\to 1}\frac{x^{20}-1}{x^{10}-1}$ **(3 marks)**

2) Solve the equation $x-xe^{5x+2}=0$ **(4 marks)**

3) Find the complex roots of the quadratic equation

$z^2-(4-i)z+(5-5i)=0$ **(4 marks)**

4) Solve the following trigonometric equation in the range given

$2\sin y+5\cos y=2\cos y, 0\le y<360^0$ **(4 marks)**

5) Prove that $\sqrt{\frac{1-\cos t}{1+\cos t}}=\frac{1-\cos t}{\sin t}$ **(3 marks)**

6) Find the equation of any horizontal tangent to $y=2x^3-24x+4$ **(3 marks)**

7) A bank advertises an interest rate of 8% per year. If you deposit 5000Frw, how much is on your account 3 years later if the interest is compounded continuously? Assume that the interest compounded continuously is modeled by $P=P_0e^{rt}$ where P_0 is the initial amount deposit on account; r is the interest rate; t time for which the amount deposited can take in the bank.

(3 marks)

8) Using De Moivre 's theorem, show that $\sin 5\theta=16\sin^5\theta-20\sin^3\theta+5\sin\theta$

(5 marks)

9) It is estimated that 50% of emails are spam emails. Some software was applied to filter these spam emails before they reach your inbox. A certain brand of software claims that it can detect 99%of spam emails, and the probability for a false positive (a non-spam email detected as spam) is 5%. Now, if an email is detected as spam, then what is the probability that it is in fact a non-spam email?

(4 marks)

10) Find the polar equation of the circle of radius 3 units and center at $(3,0)$.

(4 marks)

11) a) Explain linear dependent vectors. **(1 mark)**

b) Determine whether vectors $\vec{i}$ and $\vec{j}$ are or not linearly dependent such that $\vec{i}=(3,4)$ and $\vec{j}=(1,3)$.

(3 marks)

12) Given the equation $\frac{dy}{dx}+\frac{4y}{x}=6x-5, x>0$

Determine the solution of the above differential equation subjected to the boundary condition $y=1$ at $x=1$ **(4 marks)**

13) Given that $\frac{1}{n}\sum_{r=1}^{n}x_r=2$ and $\sqrt{\frac{1}{n}\sum_{r=1}^{n}(x_r)^2-\frac{1}{n^2}\left(\sum_{r=1}^{n}x_r\right)^2}=3$

Determine in term of n the value of $\sum_{r=1}^{n}(x_r+1)^2$ **(4 marks)**

14) Evaluate integral $\int_0^5 xe^{-x}dx$ **(3 marks)**

15)

Determine the angle between vectors $\vec{u}$ and $\vec{v}$ such that $\vec{u}=(3,4)$ and $\vec{v}=(-1,4)$. **(3 marks)**

SECTION B ATTEMPT ANY THREE QUESTIONS (45 marks)

16)

a) Given matrices $A; B$ and C such that $A=\begin{bmatrix} x+y & y \\ 2x & x-y \end{bmatrix}$; $B=\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $C=\begin{bmatrix} 3 \\ 2 \end{bmatrix}$

If $AB=C$; find the matrix A^2 **(7 marks)**

b) Find the equation of a hyperbola whose foci are $(4,2)$ and $(8,2)$ and eccentricity is 2. **(8 marks)**

17) The marks of three students in Biology and Chemistry are:

Biology (x)	5	9	13	17	21
Chemistry (y)	12	20	25	33	35

a) Find $\bar{x}$ **(2 marks)**

b) Find $\bar{y}$ **(2 marks)**

c) Calculate the covariance $\text{cov}(x,y)$ of the marks distribution in these 2 subjects. **(4 marks)**

d) Determine the standard deviations σ_x and σ_y. **(4 marks)**

e) Find the coefficient of correlation between x and y. **(3 marks)**

18) A population of bacteria initially has 250 present and in 5 days there will be 1600 bacteria present.

a) Determine the exponential growth equation for this population. **(6 marks)**

b) How long will it take for the population to grow from its initial population of 250 to a population of 2000? **(5 marks)**

c) Find an equation of the sphere whose center is $C(3,8,1)$ and passes through the point $(4,3,-1)$. **(4 marks)**

19)

a) Express $\frac{5}{(x-1)(3x+2)}$ in partial functions. **(4 marks)**

b) Hence find $\int \frac{5}{(x-1)(3x+2)}dx$, where $x>1$ **(4 marks)**

c) Find the particular solution of the differential equation:

$(x-1)(3x+2)\frac{dy}{dx}=5y, x>1$, for which $y=8$ at $x=2$.

Give your answer in the form $y=f(x)$ **(7 marks)**

20) It has been determined that the probability density function for the wait in line at a counter is given by the function:

$$f(\mathrm{t})=\begin{cases}0, \mathrm{t}<0\\ 0.1e^{\frac{-t}{10}}, t\geq 0\end{cases}$$

where t is the number of minutes spent waiting in line.

a) Verify whether the function $f(t)$ is a probability density function. **(5 marks)**

b) Determine the probability that a person will wait in line for at least 6 minutes. **(5 marks)**

c) Determine the mean wait in line. **(5 marks)**

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