MATHEMATICS

Date:20/June/2023
Period: 8:30 am-11:30 am

NESA NATIONAL EXAMINATION AND SCHOOL INSPECTION AUTHORITY
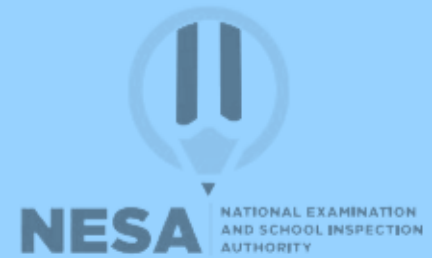
END OF TERM III EXAMINATIONS

GRADE   SENIOR TWO

OPTION   ORDINARY LEVEL

DURATION: 3 HOURS

MARKS: 100

INSTRUCTIONS

1) This paper consists of **two** section

   Section A: Attempt **all** questions.   **(55 marks)**

   Section B: Attempt **three** questions only.   **(45 marks)**

2) You may use mathematical instruments and a calculator **where necessary.**

3) Use a **blue or black ink pen only** to write your answers and a **pencil** to draw diagrams.

4) Show clearly all the working steps. **Marks will not be awarded for the answer without all working steps.**

Section A: Attempt all questions (55 marks)

1. What do you understand by the term "line of symmetry"? **(2 marks)**

2. State properties of parallel projection. **(2 marks)**

3. Rationalize the denominator: $\dfrac{\sqrt{2}}{\sqrt{5}+\sqrt{3}}$ **(4 marks)**

4. If $7a = 3b$ and $b \neq 0$, find the ratio $\dfrac{a}{b}$ **(3 marks)**

5. Given the polynomial equation

$f(y) = 2y^2 + 9y - 8$. Find $f(4)$ **(4 marks)**

6. Given the two functions $f(x) = x^2 - 1$ and $g(x) = 2x + 1$, find $f\!\circ\!g(x)$ **(3 marks)**

7. Simplify the following

$$f(x) + g(x) = (5x^4 + 3x^2 + 6x - 79) + (2x^3 - 4x^2 + 5x - 9)$$

8. Simplify the following polynomials in descending powers of x:

$$x^3 + 2xy^2 - 6x^2y + y^3 - 3xy^2 + 4x^2y \qquad \textbf{(3 marks)}$$

9. Solve the following system by substitution method

$$\begin{cases} 3x + 4y = 10 \\ 2x - 3y = 1 \end{cases}$$ **(5 marks)**
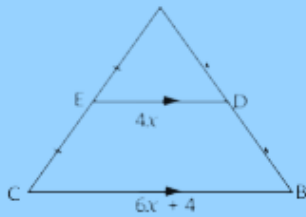
10. The result of multiplying a number by 3 and then subtracting 5 is less than multiplying the number by 2 and adding 9. Form an inequality in one unknown and solve it. **(5 marks)**

11. Use Cramer's rule to solve the equations below: **(5 marks)**

$$\begin{cases} 3x + 2y = 0 \\ x - y = -5 \end{cases}$$

12. In the triangle below, find the value of x (4 marks)

13. Given the following data 3, 4, 2, 6, 7, 8, 4, 8, 9, 2, 8, 5

    a. What is the mode of the data (1 mark)

    b. Explain your answer (2 marks)

14. Enumerate the characteristics of a vector (3 marks)

15. If $\vec{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \vec{b} = \begin{pmatrix} 3 \\ -1 \end{pmatrix},$

$\vec{c} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$, express as a single column vector:

$5\vec{a} - 4\vec{b} + \vec{c}$ (5 marks)

Section B: Attempt three questions only (45 marks)

16. The vertices of a quadrilateral are A(2, 0.5), B(2, 2), C(4, 3.5) and D(3.5,1). Find the image of the quadrilateral under reflection in line y=0 followed by reflection in line y = −x. (15 marks)

17. Given the following triangle
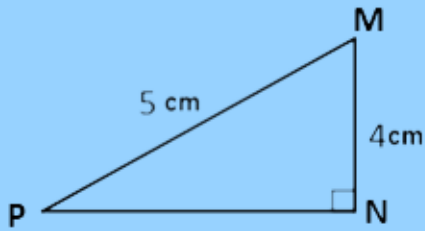
a) Determine the length $\overline{PN}$ (5 marks)

a) Calculate the perimeter (5 marks)

b) Calculate the area of the triangle NMP. (5 marks)

18. Under a translation. Given A′(0, –3), B′ (1, –5), C′(1, –2); A, B and C are points (–2, –2), (–1, –4), (–1, –1) respectively, find the translation vector.

(15 marks)

19. Calculate the arithmetic mean of the marks from the following table:

Marks	Number of students f_i
0-10	12
20-30	18
20-30	27
30-40	20
40-50	17
50-60	6

(15 marks)

20. A hat contains 3 red, 4 blue and 5 green tickets. If one ticket is chosen at random, what is the probability that it is:

  a) Red   **(5 marks)**

  b) Blue   **(5 marks)**

  c) Green   **(5 marks)**

S2 MATHS MARKING SCHEME 2023

TOTAL MARKS : 100 MARKS

ANSWER 1 2marks

A line of symmetry is a line dividing an object into two equal and exactly similar parts   **2marks**

ANSWER 2   (3 marks)

- The parallel projection on one line, all images are formed on that line.   **1 mark**
- A point on the line is mapped onto itself under parallel projection on the same line.
                                                                              1 mark
- Points are those points which lie exactly on the line of projection under parallel projection. **1mark**

If a line segment, say AB to be projected is parallel to the direction of the projection, then the two points have the same image

Answer 3   (4 marks)

$$\frac{\sqrt{2}}{\sqrt{5}-\sqrt{3}} = \frac{\sqrt{2}(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} \quad \textbf{2mark}$$

$$= \frac{\sqrt{10}+\sqrt{6}}{5-3} \quad \textbf{1mark}$$

$$= \frac{\sqrt{10}+\sqrt{6}}{2} \quad \textbf{1mark}$$

$$x^3 + 2xy^2 - 6x^2y + y^3 - 3xy^2 + 4x^2y$$

$$= x^3 + 2xy^2 - 3xy^2 - 6x^2y + 4x^2y + y^3 \quad \textbf{1 mark}$$

$$= x^3 - xy^2 - 2x^2y + y^3 \quad \textbf{1 mark}$$

$$= x^3 - 2x^2y - xy^2 + y^3 \quad \textbf{1 mark}$$

Answer 9 (5 marks)

$$\begin{cases} 3x + 4y = 10 \ (1) \\ 2x - 3y = 1 \ (2) \end{cases}$$

From (1): $x = \frac{10 - 4y}{3}$ (i)   **1 mark**

replace (i) in (2)

$$2\left(\frac{10 - 4y}{3}\right) - 3y = 1, \qquad \textbf{1 mark}$$

$$\Leftrightarrow \frac{20 - 8y}{3} - 3y = 1$$

$$\Leftrightarrow 20 - 8y - 9y \qquad \textbf{1 mark}$$
$$= 3$$

$$\Leftrightarrow -17y = 3 - 20$$

$$-17y = 3 - 20 \Leftrightarrow -17y$$
$$= -17 \Leftrightarrow y = \frac{-17}{-17} \Leftrightarrow y = 1 \qquad \textbf{1 mark}$$

$$in(i): x = \frac{10 - 4 \times 1}{3} = \frac{6}{3} = 2 \quad \textbf{1 mark}$$

$$S = \{(2,1)\}$$

ANSWER 4 (3 marks)

Divide on both side of $7a = 3b$ by $7b$ **1mark**

$$\frac{7a}{7b} = \frac{3b}{7b}$$ **1mark**

$$\frac{a}{b} = \frac{3}{7}$$ **1mark**

Answer 5 (4 marks)

$f(4) = -2(4)^2 + 9(4) - 8$ **1mark**

$f(4) = -32 + 36 - 8$ **1mark**

$f(4) = -40 + 36$ **1mark**

$f(4) = -4$ **1mark**

ANSWER 6 (3 marks)

$$fog(x) = f[g(x)]$$

$(2x+1)^2 - 1$ ....1 mark

$(2x)^2 + 2.2x.1 + 1^2$ ....1 mark

$4x^2 + 4x + 1$ ....1 mark

Answer 7   3 marks

$$= 5x^4 + 3x^2 - 4x^2 + 6x + 5x - 79 - 9 + 2x^3 \quad \textbf{1mark}$$

$$= 5x^4 - x^2 + 11x - 88 + 2x^3 \quad \textbf{1mark}$$

$$= 5x^4 + 2x^3 - x^2 + 11x - 88 \quad \textbf{1mark}$$

ANSWER 8   3 marks

Answer 10 (5 marks)

Let x be the number
Then, $3x - 5 < 2x + 9$ **1mark**
$\Rightarrow 3x - 2x < 9 + 5$ **1mark**
$\Rightarrow x < 14$ **1mark**
Any number less than 14 satisfies the given conditions. In other words, any number in the interval $]-\infty, 14[$ **2 marks**

Answer 11 (5 marks)

$D = \begin{vmatrix} 3 & 2 \\ 1 & -1 \end{vmatrix} = -5$ **1mark**

$Dx = \begin{vmatrix} 0 & 2 \\ -5 & -1 \end{vmatrix} = 10$ **1mark**

$Dy = \begin{vmatrix} 3 & 0 \\ 1 & -5 \end{vmatrix} = -15$ **1mark**

$x = \dfrac{10}{-5} = -2$ **1mark**

$y = \dfrac{-15}{-5} = 3$ **1mark**

ANSWER 12

By midpoint theorem, $DE = \dfrac{1}{2}BC$ **1mark**

$We\ get\ 4x = \dfrac{1}{2}(6x + 4)$ **1mark**

$\Leftrightarrow 8x = 6x + 4$ **1mark**

$\Leftrightarrow 8x - 6x = 4 \ldots \ldots$ **1 mark**

$\Leftrightarrow 2x = 4$

$so, x = 2$ **1mark**

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Answer 13   (3marks)

a. Mode is 8   1mark

b. Because 8 has more repetition in the set.   2 marks

Answer 14   3 marks

A vector is characterized by
- a) A line of action   **(1mark)**
- b) Direction   **(1mark)**
- c) Magnitude (size or length)   **(1mark)**

ANSWER 15 (5 marks)

$$\vec{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \vec{b} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}, \vec{c} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$$

$$5\vec{a} - 4\vec{b} + \vec{c} = 5\begin{pmatrix} 3 \\ 4 \end{pmatrix} - 4\begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} -4 \\ 0 \end{pmatrix} \quad \text{2 marks}$$

$$= \begin{pmatrix} 15 \\ 20 \end{pmatrix} - \begin{pmatrix} 12 \\ -4 \end{pmatrix} + \begin{pmatrix} -4 \\ 0 \end{pmatrix} \quad \text{2 marks}$$

$$= \begin{pmatrix} -1 \\ 24 \end{pmatrix} \quad \text{1mark}$$

ANSWER 16   15 marks

Reflection in line y= 0 followed by reflection in line y= −x means that we first obtain the image under reflection in line y=0 and then reflect this image in line y = −x.

A′(2, − 0.5)   **2 marks**
B′(2, − 2)   **2 marks**
C′(4, − 3.5)   **2 marks**
D′(3.5, 1)   **2 marks**

$A''(0.5, -2),$ 2 marks
$B''(2, -2),$ 2 marks
$C''(3.5, -4)$ 2 marks
$D''(1, 3.5)$ 1 marks

ANSWER 17

$MN^2 + NP^2 = MP^2$ 1mark
$4^2 + NP^2 = 5^2$ 2marks
$NP^2 = 5^2 - 4^2$ 1 mark
$NP^2 = 25 - 16$ 1 mark
$NP^2 = 9$ 2marks
$NP = \sqrt{9}\ cm = 3\ cm$ 2 marks
Perimeter $= MN + NP + MP = 5cm + 4cm + 3cm = 12\ cm$ **(4 marks)**
Area $= \frac{NP \times NM}{2} = \frac{3 \times 4}{2} = 6cm^2$ **0.5marks**   **(2 marks)**

ANSWER 18

Suppose the translation vector is $\binom{x}{y}$,..........................I mark

$OA + \binom{x}{y} = OA'$....................... 2

$\therefore \Rightarrow \binom{x}{y} = OA' - OA$ 2

$\Rightarrow \binom{-2}{-2} + \binom{x}{y} = \binom{0}{3}$ 3

$\binom{x}{y} = \binom{0}{-3} - \binom{-2}{-2} = \binom{2}{-1}$ 3

Similarly, we can use points B and B'to find $\binom{x}{y}$

thus,

$\binom{x}{y} = \binom{1}{-5} - \binom{-1}{-4} = \binom{2}{-1}$ ....2marks   *and* $\binom{x}{y} = \binom{1}{-2} - \binom{-1}{-1} =$
$\binom{2}{-1}$ **1mark**

The translation vector is $\binom{2}{-1}$ ... ... **1 mark**

Answer 19

Here maximum frequency is 28. Thus the class 40-50 is the modal class. **1mark**

Using the formula for continuous, the value of mode is given by

$$Mode = 39.5 + \frac{10(28-12)}{(28-12)+(28-20)} \quad \textbf{2marks}$$

$$= 39.5 + \frac{160}{24} \quad \textbf{1mark}$$
$$= 46.167 \quad \textbf{1mark}$$

Marks	Number of students f_i	Mid-point x_i	$(f_i x_i)$	
0-10	12	5	60	**1mark**
20-30	18	15	270	**1mark**
20-30	27	25	675	**1mark**
30-40	20	35	700	**1mark**
40-50	17	45	765	**1mark**
50-60	6	55	330	**1mark**
Total	100		2800	**1mark**

$$\bar{x} = \frac{1}{n}\sum fx = \frac{1}{100} \times 2800 = 28 \quad \textbf{3marks}$$

Answer 20

Total number of the tickets: (3+4+5) = 12

$$a) p(Red) = \frac{3}{12} = \frac{1}{4} = 0.25 \ldots \ldots \ldots 5 \; marks$$

$$b) p(blue) = \frac{4}{12} = \frac{1}{3} = 0.33 \ldots \ldots \ldots 5 \; marks$$

$$c) p(Green) = \frac{5}{12} = 0.416 \ldots \ldots \ldots 5 \; marks$$

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