

MATHEMATICS II

Date: 20/06/2024

Period: 08.30 – 11.30



END OF TERM III EXAMINATIONS QUESTION PAPER

GRADE: SENIOR FIVE

COMBINATIONS: MATHEMATICS-CHEMISTRY-BIOLOGY (**MCB**)

-- MATHEMATICS -COMPUTER SCIENCE-
ECONOMICS (**MCE**)

- MATHEMATICS-ECONOMICS-GEOGRAPHY
(**MEG**)

- MATHEMATICS -PHYSICS-COMPUTER
SCIENCE (**MPC**)

- MATHEMATICS-PHYSICS-GEOGRAPHY (**MPG**)

- PHYSICS-CHEMISTRY-MATHEMATICS (**PCM**)

DURATION: 3 HOURS

MARKS:

100

CAMIS

...../70

INSTRUCTIONS

1) This paper contains **two** sections:

Section A: Attempt **all** questions. **(55 marks)**

Section B: Attempt **three** questions only. **(45 marks)**

2) You may use mathematical instruments and a calculator **where necessary**.

3) Use a **blue or black ink pen only** to write your answers and a **pencil** to draw diagrams.

4) Show clearly all the working steps. **Marks will not be awarded for the answer without all working steps.**

Section A: Attempt all questions (55 marks)

1. Show that $\cos 3A = 4\cos^3 A - 3\cos A$ **(4 marks)**

2. If $\sin A = \frac{3}{5}$, $0 < A < \frac{\pi}{2}$ and $\sin B = \frac{4}{5}$, $\frac{\pi}{2} < B < \pi$ Evaluate $\cos(A+B)$ without the use of a calculator. **(4 marks)**

3. Solve the trigonometric equation $2\sin \frac{x}{2} - \cos x + 1 = 1$ **(5 marks)**

4. Show that $U_n = \{3(-2)^n\}$ is a geometric sequence and write down the first 4 terms of this sequence. **(3 marks)**

5. The population of a country grow according to the law $p = Ae^{0.06t}$, where p is million in the population at time t and A is a constant given that t=0 the population is 27.3 millions

Calculate the population when :

i. t=10 **(2 marks)**

ii. t=15 **(2 marks)**

6. Fill in blanks $y = \sin^{-1}x$ means that $x = \dots\dots\dots$ where $-1 \leq x \leq 1$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$ **(2 marks)**

7. Calculate $\lim_{x \rightarrow 0} \frac{\sin 8x}{\sin 5x}$ **(3 marks)**

8. Study the parity of the function $f(x) = \frac{x + \sin 3x}{x^2}$ **(4 marks)**

9. Answer by **True** or **False**. Two vectors are parallel if:

a) They have the opposite directions **(1 mark)**

b) They have the same magnitude **(1 mark)**

c) They lie in the same plane **(1 mark)**

10. a) Show that $B = \{\vec{u}, \vec{v}, \vec{w}\}$ where $\vec{u} = (1, -1, 2)$, $\vec{v} = (1, 1, 1)$, $\vec{w} = (-1, 0, 1)$ is a basis of \mathbb{R}^3 . **(2 marks)**

b) Find the components of the vector $\vec{t} = (0, -3, 4)$ in the basis B **(2 marks)**

11. Without calculating explain why $\Delta = \begin{vmatrix} 1 & 5 & -2 \\ 2 & 10 & -4 \\ 3 & 1 & 5 \end{vmatrix} = 0$ **(2 marks)**

12. Given A and B matrices representation of linear transformation f and g respectively such that

$$A = \begin{pmatrix} 3 & 4 & 1 \\ -1 & 2 & 0 \\ 4 & -5 & -3 \end{pmatrix} \text{ and } B = \begin{pmatrix} 2 & 3 & 2 \\ -1 & 0 & 3 \\ 1 & 0 & 2 \end{pmatrix}.$$

Find the matrix representation of $g \circ f$. **(3 marks)**

13. Find the centre and the radius of the Sphere

$$S \equiv x^2 + y^2 + z^2 + 4x - 8y + 6z - 7 = 0$$

(4 marks)

14. Find the Cartesian equation of the plane α through the points $a(3, 2, -1)$; $b(4, 4, 0)$ and perpendicular to the plane $\beta \equiv 2x + 4y - 4z = 3$

(5 marks)

15. Find the vector position of the point of intersection of the lines

$$L_1 \equiv \begin{cases} X = 3 + 3t \\ y = 3 + 2t \\ z = t \end{cases} \text{ and } L_2 \equiv \begin{cases} X = 4 + s \\ y = 3 + s \\ z = -1 + s \end{cases} \quad \textbf{(5 marks)}$$

Section B: Attempt any three questions only (45 marks).

16. The first 3 terms of an arithmetic series are $k-2$, $2k+5$ and $4k+1$.

a) Show that $k=11$ **(3 marks)**

b) Find the 41th term of the series **(7 marks)**

c) Show that S_n is always a square number **(5 marks)**

17. Using the iterative formula, show that the 4th root of the number N is

$$x_{n+1} = \frac{3}{4}x_n + \frac{N}{4x_n^3}. \text{ Hence show that } (45.7)^{1/4} \approx 2.600 \text{ (correct to 3dps) } \quad \textbf{(15 marks)}$$

18. If $A = \begin{pmatrix} \cos A & -\sin A & 0 \\ \sin A & \cos A & 0 \\ 0 & 0 & 1 \end{pmatrix}$ verify that $A \cdot (\text{Adj}) = \det(A) \cdot I$ **(15 marks)**

19. Let consider the following table

x_i	-5	-1	3	10	13
y_i	33	25	17	3	-3

- i) Calculate the coefficient of correlation and comment on it. **(8 marks)**
 ii) Find the equation of regression line y on x . **(5 marks)**
 iii) Estimate the value of x when $y=16$. **(2 marks)**

20. a) If A and B are dependent events such that $p(A) = \frac{5}{8}$ and $p(B/A) = \frac{3}{7}$

Find $p(A \cap B)$ **(5 marks)**

b) 3 girls A, B, and C pack biscuit in a factory from the batch allocated to them A packs 55%, B packs 30% and C packs 15%. The probability that A break some biscuits in packet is 0.7 and the respective probability of B is 0.2 and C is 0.1. What is the probability that a packet with broken biscuits found by checker was packed by A. **(10 marks)**

END!!!!

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END OF TERM III EXAMINATIONS MARKING GUIDE

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SENIOR FIVE

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SECTION A: ATTEMPT ALL QUESTIONS (55 MARKS)

1. $\cos 3A = 4 \cos^3 A - 3 \cos A$

We know that:

$$\begin{aligned} \cos 3A &= \cos(2A + A) && \dots\dots\dots 0.5 \\ &= \cos 2A \cos A - \sin 2A \sin A && \dots\dots\dots 1 \\ &= (\cos^2 A - \sin^2 A) \cos A - 2 \sin^2 A \cos A && \dots\dots\dots 0.5 \\ &= (\cos^2 A - (1 - \cos^2 A)) \cos A - 2(1 - \cos^2 A) \cos A && \dots\dots\dots 0.5 \\ &= (\cos^2 A - 1 + \cos^2 A) \cos A - 2(1 - \cos^2 A) \cos A && \dots\dots\dots 0.5 \\ &= \cos^3 A - \cos A + \cos^3 A - 2 \cos A + 2 \cos^3 A && \dots\dots\dots 0.5 \\ &= 4 \cos^3 A - 3 \cos A \quad \text{as required} && \dots\dots\dots 0.5 \end{aligned}$$

2. $\sin A = \frac{3}{5} \quad 0 < A < \frac{\pi}{2}$
 $\sin B = \frac{4}{5} \quad \frac{\pi}{2} < B < \pi$

$$\begin{aligned} \cos(A + B) &= \cos A \cos B - \sin A \sin B && \dots\dots\dots 1 \\ \cos A &= \pm \sqrt{1 - \sin^2 A} = \pm \sqrt{1 - \frac{9}{25}} = \frac{4}{5}, \quad 0 < A < \frac{\pi}{2} && \dots\dots\dots 1 \\ \cos B &= \pm \sqrt{1 - \sin^2 B} = \pm \sqrt{1 - \frac{16}{25}} = -\frac{3}{5}, \quad \frac{\pi}{2} < B < \pi && \dots\dots\dots 1 \end{aligned}$$

Thus

$$\begin{aligned} \cos A \cos B - \sin A \sin B &= \frac{4}{5} \left(-\frac{3}{5} \right) - \frac{3}{5} \left(\frac{4}{5} \right) && \dots\dots\dots 0.5 \\ &= -\frac{24}{25} && \dots\dots\dots 0.5 \end{aligned}$$

3. $2 \sin^2 \frac{x}{2} - \cos x + 1 = 1$

$$2 \sin^2 \frac{x}{2} + (1 - \cos x) = 1 \dots\dots\dots 0.5$$

$$2 \sin^2 \frac{x}{2} + 2 \sin^2 \frac{x}{2} = 1 \dots\dots\dots 0.5$$

$$4 \sin^2 \frac{x}{2} = 1 \dots\dots\dots 0.5$$

$$\sin^2 \frac{x}{2} = \frac{1}{4}$$

$$\Rightarrow \sin \frac{x}{2} = \pm \frac{1}{2} \dots\dots\dots 0.5$$

For $\sin \frac{x}{2} = \frac{1}{2}$

$$\frac{x}{2} = \begin{cases} \frac{\pi}{6} + 2k\pi \\ \frac{5\pi}{6} + 2k\pi \end{cases} \quad 0.5 \Rightarrow x = \begin{cases} \frac{\pi}{3} + 4k\pi \\ \frac{5\pi}{3} + 4k\pi \end{cases}, k \in \mathbb{Z} \dots\dots\dots 1$$

For $\sin \frac{x}{2} = -\frac{1}{2}$

$$\frac{x}{2} = \begin{cases} \frac{4\pi}{3} + 2k\pi \\ \frac{5\pi}{3} + 2k\pi \end{cases} \quad 0.5 \Rightarrow x = \begin{cases} \frac{8\pi}{3} + 4k\pi \\ \frac{10\pi}{3} + 4k\pi \end{cases}, k \in \mathbb{Z} \dots\dots\dots 1$$

$$\mathbb{S} = \left\{ \frac{\pi}{3} + 4k\pi, \frac{5\pi}{3} + 4k\pi, \frac{8\pi}{3} + 4k\pi, \frac{10\pi}{3} + 4k\pi \right\} \dots\dots\dots 1$$

4. $u_n = \{3(-2)^n\}$

$$u_{n+1} = 3(-2)^{n+1} \dots\dots\dots 1$$

$$\frac{u_{n+1}}{u_n} = \frac{3(-2)^{n+1}}{3(-2)^n}$$

$$\dots\dots\dots 1$$

$$= \frac{-2(-2)^n}{(-2)^n} \dots\dots\dots 0.5$$

$$= -2 \dots\dots\dots 0.5$$

5. i. $p = Ae^{0.06t}$ 0.5
 $p(t=0) = Ae^{0.06 \times 0} = 27.3$ 0.5
 $\Rightarrow A = 27.3$ Then $p = 27.3e^{0.06t}$ 0.5
for $t=10$, $p = 27.3e^{0.06 \times 10} = 49.7$ millions0.5
- ii. for $t=15$, $p = 27.3e^{0.06 \times 15} = 67.1$ millions2

6. $x = \sin y$ 2

7. $\lim_{x \rightarrow 0} \frac{\sin 8x}{\sin 5x} = \frac{8}{5} \lim_{x \rightarrow 0} \frac{5x \sin 8x}{8x \sin 5x}$ 1
 $= \lim_{x \rightarrow 0} \frac{\sin 8x}{8x} \cdot \lim_{x \rightarrow 0} \frac{5x}{\sin 5x}$ 1
 $= \frac{8}{5}$ 1

8. $Dom f = \mathbb{R} \setminus \{0\}$ 0.5

$\forall x \in Dom f \Rightarrow -x \in Dom f$ 0.5

$f(-x) = \frac{-(x) + \sin 3(-x)}{(-x)^2}$ 0.5

$= \frac{-x - \sin 3x}{x^2}$ 1

$= -\frac{x + \sin 3x}{x^2} = -f(x)$ 0.5

Thus $f(x)$ is an odd function1

9. a) True1

b) False1

c) False1

10. a) Since $\dim \mathbb{R}^3 = 3$, to show that 3 vectors form a basis it is sufficient to

show that the 3 vectors are linearly independent0.5

$$\det(\vec{u}, \vec{v}, \vec{w}) = \begin{vmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & 1 & 1 \end{vmatrix} = 1 + 2 + 2 = 5 \neq 0 \dots\dots\dots 0.5$$

Therefore $\vec{u}, \vec{v}, \vec{w}$ are linearly independent $(0, -3, 4) = x(1, -1, 2) + y(1, 1, 1) + z(-1, 0, 1)$
independent, hence they form a basis of \mathbb{R}^3 1

b) $R_2 = 2R_1$ Let $\vec{t} = x\vec{u} + y\vec{v} + z\vec{w} \Leftrightarrow (0, -3, 4) = x(1, -1, 2) + y(1, 1, 1) + z(-1, 0, 1)$

$$\begin{cases} x + y - z = 0 \\ -x + y = -3 \\ 2x + y + z = 4 \end{cases} \Leftrightarrow x = 2, y = -1, z = 1 \dots\dots\dots 1$$

The components of \vec{t} are $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ in the basis B 1

11. As the second row is twice the first row i.e $R_2 = 2R_1$ then $\Delta = 0$ 2

12. $gof = \begin{pmatrix} 2 & 3 & 2 \\ -1 & 0 & 3 \\ 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} 3 & 4 & 1 \\ -1 & 2 & 0 \\ 4 & -5 & -3 \end{pmatrix} \dots\dots\dots 1$

$$= \begin{pmatrix} 6-3+8 & 8+6-10 & 2+0-6 \\ -3+0+12 & -4+0-15 & -1+0-9 \\ 3+0+8 & 4+0-10 & 1+0-6 \end{pmatrix} \dots\dots\dots 1$$

$$= \begin{pmatrix} 11 & 4 & -4 \\ 9 & -19 & -10 \\ 11 & -6 & -5 \end{pmatrix} \dots\dots\dots 1$$

13. $S \equiv x^2 + y^2 + z^2 + 4x - 8y + 6z - 7 = 0$

$$\equiv x^2 + 4x + y^2 - 8y + z^2 + 6z - 7 = 0 \dots\dots\dots 1$$

$$\equiv (x+2)^2 + (y-4)^2 + (z+3)^2 - 4 - 16 - 9 - 7 = 0 \dots\dots\dots 1$$

$$\equiv (x+2)^2 + (y-4)^2 + (z+3)^2 = 36 \dots\dots\dots 1$$

Hence this is a circle with center $C(-2, 4, -3)$ and Radius $R = 6 \dots\dots\dots 1$

14. Given that $a(3, 2, -1)$, $b(4, 4, 0)$, $B \equiv 2x + 4y - 4z = 3$

$$\overline{ab} = (1, 2, 1)$$

$$\dots\dots\dots 0.5$$

$$B^\perp = (2, 4, -4) \dots\dots\dots 0.5$$

$$\alpha = (a, \overline{ab}, B^\perp) \dots\dots\dots 1$$

$$\alpha \equiv \begin{vmatrix} x-3 & 1 & 2 \\ y-2 & 2 & 4 \\ z+1 & 1 & -4 \end{vmatrix} = 0 \dots\dots\dots 1$$

$$\alpha \equiv -12(x-3) + 6(y-2) + 0(z+1) = 0 \dots\dots\dots 1$$

$$\alpha \equiv -12x + 36 + 6y - 12 + 0z = 0 \dots\dots\dots 0.5$$

$$\alpha \equiv 2x - y - 12 = 0 \dots\dots\dots 0.5$$

$$15. L_1 \cap L_2 = \begin{cases} 3 + 3t = 4 + \lambda \\ 3 + 2t = 3 + \lambda \\ t = -1 + \lambda \end{cases} \Rightarrow \begin{cases} 3t - \lambda = 1 \\ 2t - \lambda = 0 \\ t - \lambda = -1 \end{cases} \dots\dots\dots 1$$

$$\begin{cases} 3t - \lambda = 1 \\ 2t - \lambda = 0 \end{cases} \begin{vmatrix} 1 \\ -1 \end{vmatrix} \Rightarrow \begin{cases} t = 1 \\ \lambda = 2 \end{cases} \dots\dots\dots 1$$

$$\text{for } t = 1, x = 6, y = 5, z = 1 \dots\dots\dots 1$$

$$L_1 \cap L_2 = \{(6, 5, 1)\} \dots\dots\dots 1$$

$$\text{Vector position } \vec{V} = 6\vec{i} + 5\vec{j} + \vec{k} \dots\dots\dots 1$$

SECTION B: ATTEMPT ANY THREE QUESTIONS ONLY (45 MARKS)

16. Given that $k - 2$, $2k + 5$ and $4k + 1$ are terms of an a.s

a) $2k + 5 = \frac{k - 2 + 4k + 1}{2}$ 1

$\Leftrightarrow 4k + 10 = 5k - 1$ 1

$\Leftrightarrow 5k - 4k = 11$ 0.5

Hence $k = 11$ as required0.5

b) $u_n = u_1 + (n - 1)d$ 1

$u_1 = 11 - 2 = 9$ 1

$u_2 = 2(11) + 5 = 27$ 1

$u_3 = 4(11) + 1 = 45$ 1

The common difference $d = 18$ 1

$u_{41} = u_1 + 40d$ 1

$= 9 + 40(18) = 729$ 1

c) $S_n = \frac{n}{2}(u_1 + u_n)$

$= \frac{n}{2}(u_1 + u_1 + (n - 1)d)$ 1

$= \frac{n}{2}(2u_1 + (n - 1)d)$ 1

$= nu_1 + \frac{n(n - 1)}{2}d$ 0.5

$$\begin{aligned}
&= 9n + \frac{n(n-1)}{2} \cdot 18 \dots\dots\dots 0.5 \\
&= 9n + 9n^2 - 9n \dots\dots\dots 0.5 \\
&= 9n^2 \dots\dots\dots 0.5 \\
&= (3n)^2 \quad \text{which is always square.} \dots\dots\dots 1
\end{aligned}$$

17. Using Newton Raphson method:

$$\begin{aligned}
\text{let } X &= \sqrt[4]{N} \dots\dots\dots 1 \\
\Rightarrow X^4 &= N \dots\dots\dots 0.5 \\
X^4 - N &= 0 \dots\dots\dots 1 \\
\Rightarrow f(X) &= X^4 - N \dots\dots\dots 1 \\
X_{n+1} &= X_n - \frac{f(X_n)}{f'(X_n)} \quad \text{for } n \in \mathbb{N} \dots\dots\dots 1 \\
f'(X) &= 4X^3 \dots\dots\dots 1 \\
X_{n+1} &= X_n - \left(\frac{X_n^4 - N}{4X_n^3} \right) \dots\dots\dots 1 \\
&= \frac{4X_n^4 - X_n^4 + N}{4X_n^3} \dots\dots\dots 1 \\
&= \frac{3X_n^4 + N}{4X_n^3} \dots\dots\dots 1 \\
&= \frac{3}{4} X_n + \frac{N}{4X_n^3} \dots\dots\dots 1 \\
\text{for } X &= \sqrt[4]{45.7}, \quad f(x) = X^4 - 45.7 \dots\dots\dots 1 \\
f(2) &= 16 - 45.7 < 0 \text{ and } f(3) = 81 - 45.7 > 0 \text{ hence } x \in [2,3]
\end{aligned}$$

$$X_0 = \frac{2+3}{2} = 2.5 \quad \dots\dots\dots 1$$

$$X_1 = \frac{3}{4}X_0 + \frac{N}{4X_0^3} \quad \dots\dots\dots 1$$

$$= \frac{3}{4}(2.5) + \frac{45.7}{4(2.5)^3} = 2.6062\dots \quad \dots\dots\dots 1$$

$$X_1 = \frac{3}{4}X_1 + \frac{N}{4X_1^3} \quad \dots\dots\dots 0.5$$

$$= \frac{3}{4}(2.6062) + \frac{45.7}{4(2.6062)^3} = 2.60006\dots \quad \dots\dots\dots 0.5$$

$$X_2 \cong 2.600061 \quad \dots\dots\dots 0.5$$

$$18. \det(A) = \begin{vmatrix} \cos A & -\sin A & 0 \\ \sin A & \cos A & 0 \\ 0 & 0 & 1 \end{vmatrix} \quad \dots\dots\dots 0.5$$

$$= \cos A \begin{vmatrix} \cos A & 0 \\ 0 & 1 \end{vmatrix} - \sin A \begin{vmatrix} -\sin A & 0 \\ 0 & 1 \end{vmatrix} \quad \dots\dots\dots 0.5$$

$$= \cos^2 A + \sin^2 A = 1 \quad \dots\dots\dots 0.5$$

Now let compute the 9 cofactors

$$A_{11} = \begin{vmatrix} \cos A & 0 \\ 0 & 1 \end{vmatrix} = \cos A \quad \dots\dots\dots 1$$

$$A_{22} = \begin{vmatrix} \cos A & 0 \\ 0 & 1 \end{vmatrix} = \cos A \quad \dots\dots\dots 1$$

$$A_{21} = \begin{vmatrix} -\sin A & 0 \\ \cos A & 1 \end{vmatrix} = -\sin A \quad \dots\dots\dots 1$$

$$A_{12} = \begin{vmatrix} -\sin A & 0 \\ 0 & 1 \end{vmatrix} = -\sin A \quad \dots\dots\dots 1$$

$$A_{31} = \begin{vmatrix} -\sin A & 0 \\ \cos A & 0 \end{vmatrix} = 0 \quad \dots\dots\dots 1$$

$$A_{32} = - \begin{vmatrix} \cos A & 0 \\ \sin A & 0 \end{vmatrix} = 0 \quad \dots\dots\dots 1$$

$$A_{13} = \begin{vmatrix} \sin A & \cos A \\ 0 & 0 \end{vmatrix} = 0 \quad \dots\dots\dots 1$$

$$A_{23} = - \begin{vmatrix} \cos A & -\sin A \\ 0 & 0 \end{vmatrix} = 0 \quad \dots\dots\dots 1$$

$$A_{33} = \begin{vmatrix} \cos A & -\sin A \\ \sin A & \cos A \end{vmatrix} = \cos^2 A + \sin^2 A = 1 \text{ bb} \dots\dots\dots 1$$

$$Adj = \begin{pmatrix} \cos A & \sin A & 0 \\ -\sin A & \cos A & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \dots\dots\dots 1$$

$$A \cdot Adj = \begin{pmatrix} \cos A & -\sin A & 0 \\ \sin A & \cos A & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos A & \sin A & 0 \\ -\sin A & \cos A & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \dots\dots\dots 0.5$$

$$= \begin{pmatrix} \cos^2 A + \sin^2 A & 0 & 0 \\ 0 & \cos^2 A + \sin^2 A & 0 \\ 0 & 0 & \cos^2 A + \sin^2 A \end{pmatrix} \quad \dots\dots\dots 0.5$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \dots\dots\dots 0.5$$

$$\text{also } \det(A)I = 1 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \dots\dots\dots 1$$

hence $A \cdot Adj = \det(A)I$
 1

19. i)

x_i	y_i	$x_i y_i$	x_i^2	y_i^2
-5	33	-165	25	1089
-1	25	-25	1	625
3	17	51	9	289
10	3	30	100	9
13	-3	-39	169	9
$\sum x_i =$	$\sum y_i =$	$\sum x_i y_i =$	$\sum x_i^2 =$	$\sum y_i^2 =$
20	75	-148	304	2021

.....0.5 marks for each row = 3 marks (considering last six rows)

$$\bar{x} = \frac{\sum x_i}{n} = \frac{20}{5} = 4, \bar{y} = \frac{\sum y_i}{n} = \frac{75}{5} = 15 \dots\dots\dots 1$$

$$r = \frac{n \sum x_i y_i - \sum x_i \sum y_i}{\sqrt{n \sum x_i^2 - (\sum x_i)^2} \sqrt{n \sum y_i^2 - (\sum y_i)^2}} \dots\dots\dots 1$$

$$= \frac{5(-148) - 20(75)}{\sqrt{5(304) - (20)^2} \sqrt{5(2041) - (75)^2}} \dots\dots\dots 1$$

$$\bar{x} = \frac{-740 - 1500}{\sqrt{1120} \sqrt{4480}} \dots\dots\dots 1$$

$$= \frac{-2240}{2240} \dots\dots\dots 0.5$$

$$= -1 \dots\dots\dots 0.5$$

Hence $r = -1$, there is a negative perfect correlation. The points are on straight line.

$$ii) \quad a = \frac{n \sum x_i y - \sum x_i \sum y_i}{n \sum x_i^2 - (\sum x_i)^2} \dots\dots\dots 1$$

$$= \frac{5(-148) - 20(75)}{5(304) - (20)^2} \dots\dots\dots 1$$

$$= \frac{-740 - 1500}{1120} \dots\dots\dots 0.5$$

$$= \frac{-2240}{1120} = -2 \dots\dots\dots 0.5$$

$$b = \bar{y} - a\bar{x} \dots\dots\dots 1$$

$$= 15 + 2(4) = 23 \dots\dots\dots 0.5$$

The regression $y = -2x + 23$ 0.5

iii) When $y = 16$

$$16 = -2x + 23 \Rightarrow 2x = 23 - 16 \dots\dots\dots 1$$

$$x = \frac{7}{2} = 3.5 \dots\dots\dots 1$$

20. a) Given that $p(B / A) = \frac{5}{8}$, $p(A) = \frac{3}{7}$ 1

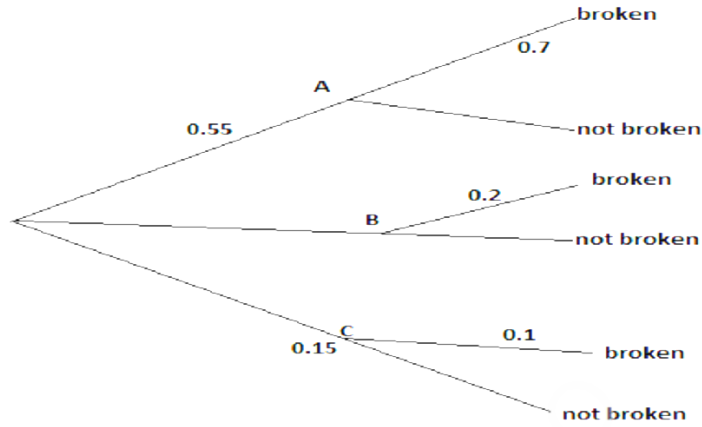
$$p(B / A) = \frac{p(A \cap B)}{p(A)} \dots\dots\dots 2$$

$$\Rightarrow p(A \cap B) = p(B / A)p(A) \dots\dots\dots 1$$

$$= \frac{5}{8} \cdot \frac{3}{7} \dots\dots\dots 1$$

$$= \frac{15}{56}$$

b)



..... 1 mark for each line = 6 marks

$$p(\text{broken}) = p(A \cap \text{broken}) + p(B \cap \text{broken}) + p(C \cap \text{broken}) \dots\dots\dots 1$$

$$= 0.55 \times 0.7 + 0.30 \times 0.2 + 0.15 \times 0.1 \dots\dots\dots 0.5$$

$$= 0.385 + 0.06 + 0.015 = 0.46 \dots\dots\dots 0.5$$

$$p(A / \text{broken}) = \frac{p(A \cap \text{broken})}{p(\text{broken})} \dots\dots\dots 1$$

$$= \frac{0.385}{0.46} \dots\dots\dots 0.5$$

$$p(A / \text{broken}) = 0.836 \dots\dots\dots 0.5$$