MATHEMATICS II

Date: 20/06/2024
Period: 08.30 – 11.30

NESA NATIONAL EXAMINATION AND SCHOOL INSPECTION AUTHORITY

END OF TERM III EXAMINATIONS
QUESTION PAPER

GRADE: **SENIOR FOUR**

COMBINATIONS:

- MATHEMATICS-CHEMISTRY-BIOLOGY **(MCB)**
- MATHEMATICS -COMPUTER SCIENCE-ECONOMICS **(MCE)**
- MATHEMATICS-ECONOMICS-GEOGRAPHY **(MEG)**
- MATHEMATICS -PHYSICS-COMPUTER SCIENCE **(MPC)**
- MATHEMATICS-PHYSICS-GEOGRAPHY **(MPG)**
- PHYSICS-CHEMISTRY-MATHEMATICS **(PCM)**

DURATION: **3 HOURS**

MARKS: **100** **CAMIS** **...../70**

INSTRUCTIONS

1) This paper contains **two** sections:

Section A: Attempt **all** questions. **(55 marks)**

Section B: Attempt **three** questions only. **(45 marks)**

2) You may use mathematical instruments and a calculator **where necessary.**

3) Use a **blue or black ink pen only** to write your answers and a **pencil** to draw diagrams.

4) Show clearly all the working steps. **Marks will not be awarded for the answer without all working steps.**

Section A: Attempt all questions (55 marks)

Find in degrees to 1 decimal place, the size of the angles in the triangle ABC such that $< CAB$ and $< ACB$ where AC=4cm, BC=5cm and angle $< ABC = 42$cm

(3 marks)

1. Prove that $\frac{\tan^2 x+2}{1+\tan^2 x} = 1 + \cos^2 x$ **(5 marks)**
2. What is Tautology? **(2 marks)**
3. Rationalize the denominator and simplify the expression $\frac{2\sqrt{6}+5\sqrt{32}}{-1-2\sqrt{3}}$

(3 marks)

4. List at least four properties of power. **(4 marks)**
5. Find the value of x if $logx = \frac{2}{3}log27 - log9 + \frac{1}{4}log\,81$ **(4 marks)**
6. Solve the following inequality
   $|2x-5| > |3-x|$ **(5 marks)**
7. Solve in set of real number the equation $2x^2+12x=-16$

(4 marks)

8. Find the domain of definition for the following function $f(x)=\frac{x^2-2}{x^2-8x+15}$

(4 marks)

9. Find the inverse of function $f(x)=x^2-8$ **(3 marks)**
10. Evaluate the limit of the function $f(x)=\lim_{x\to 2}\frac{x^3+x^2-4x-4}{x-2}$

(4 marks)

11. Find and classify the stationary points of the function $f(x)$ by using first and second derivative test such that $f(x)=x^2-4x+5$

(3 marks)

12. Write the vector equation $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + t\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ in the form $ax+by=c$

(4 marks)

13. Find the coordinates of the midpoints of the straight line joining the following pairs of point: $(3,7),(5,9)$. **(4 marks)**
14. A group of 20 university students contains eight who are in their first year of study. A student is picked at random to present the group at

a meeting. Find the probability that the student is not in the first year of study. **(3 marks)**

Section B: Attempt any three questions only (45 marks)

16. The operation * is defined in set of real numbers by $a*b=a+b-ab$

a) Calculate $(-2)*(4*1)$ and $(-2*4)*1$ and give the conclusion . **(7 marks)**

b) Find the identity element. **(3 marks)**

c) Determine the inverse element under this operation and hence calculate the inverse of -3. **(5 marks)**

17. Given the function $f:\mathbb{R}\to\mathbb{R}:x\to\frac{x^3}{x^2-1}$

a) Find the domain of definition. **(2 marks)**

b) Calculate the limits on boundaries of domain of definition and deduce the asymptote equations. **(6 marks)**

c)Is the function f odd or even? **(2 marks)**

d) Perform the variations f and determine the concavity of the curve representing the function f. **(3 marks)**

e) Make the table of variation representing the function f **(2 marks)**

18. Consider the matrices

$A=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix}$ *and* $B=\begin{pmatrix}2 & -3\\ -1 & 5\end{pmatrix}$

a) Evaluate both $(A+B)^2$ *and* $A^2+AB+BA+B^2$ **(10 marks)**

b) Explain why $(A+B)^2 \neq A^2+2AB+B^2$ **(2 marks)**

c) Ssuggest the correct expansion for $(A+B)^2$ **(3 marks)**

19. a) Complete the following table showing marks of students in a Physics Test out of 10marks **(8 marks)**

x	f	x^2	fx	fx^2
3	2			
4	3			
5	5			
6	1			
7	2			
8	2			
9	6			
$\sum$				

b) find:

(i) the mean of the marks **(2 marks)**

(ii) the variance of the marks **(3 marks)**

(iii) the standard deviation of the marks **(2 marks)**

20. A committee of 5 students is to be chosen from 6 boys and 9 girls. Find the number of ways that this can be done if:

a) There are no restrictions. **(2 marks)**

b) There are to be 3 boys and 2 girls on the committee. **(3 marks)**

c) There must be more girls than boys on the committee. **(10 marks)**

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MARKING GUIDE

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SECTION A: ATTEMPT ALL QUESTIONS (55 MARKS)

1. By using sine law $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ 0.5

$\frac{5}{\sin A} = \frac{4}{\sin 42}$ 0.5

$\sin A = \frac{5 \sin 42}{4}$ o.5

$A = \sin^{-1}\left(\frac{5 \sin 42}{4}\right) = 56.7^0$ 0.5

$< CAB = 56.7^0$ 0.5

$< ACB = 180^0 - 56.7^0 - 42^0 = 81.3^0$ 0.5

2. $\frac{\tan^2 x+2}{1+\tan^2 x} = 1 + \cos^2 x$

$\frac{\tan^2 x+1+1}{1+\tan^2 x} = 1 + \cos^2 x$ 2

$\frac{\sec^2 x+1}{\sec^2 x} = 1 + \cos^2 x$ 1

$\frac{\sec^2 x}{\sec^2 x} + \frac{1}{\sec^2 x} = 1 + \cos^2 x$ 1

$1 + \cos^2 x = 1 + \cos^2 x$ proved 1

3. Tautology s a compound proposition that is true for all possible truth values of its components.....2

4. $\frac{2\sqrt{6}+5\sqrt{32}}{-1-2\sqrt{3}} = \frac{\left(2\sqrt{6}+5\sqrt{32}\right)\left(-1+2\sqrt{3}\right)}{\left(-1-2\sqrt{3}\right)\left(-1+2\sqrt{3}\right)}$ 1

$= \frac{-2\sqrt{6}+4\sqrt{18}-5\sqrt{32}+10\sqrt{96}}{1-12}$ 1

$= \frac{-2\sqrt{6}+12\sqrt{2}-20\sqrt{2}+40\sqrt{6}}{-11}$ 0.5

$= \frac{38\sqrt{6}-8\sqrt{2}}{-11}$ .....0.5 marks

5. a) $a^n \times a^m = a^{n+m}$ 1

b) $\frac{a^n}{a^m} = a^{n-m}$ .....q.....1

c) $(ab)^n = a^n b^n$ 1

d) $(a^n)^m = a^{nm}$ 1

e) $\frac{1}{a^n} = a^{-n}$

6. $\log x = \frac{2}{3}\log 27 - \log 9 + \frac{1}{4}\log 81$

$= \frac{2}{3}\log 3^3 - \log 3^2 + \frac{1}{4}\log 3^4$ 1

$= 3\frac{2}{3}\log 3 - 2\log 3 + 4\frac{1}{4}\log 3$ 1

$= 2\log 3 - 2\log 3 + \log 3$ 1

$= \log 3$ 0.5

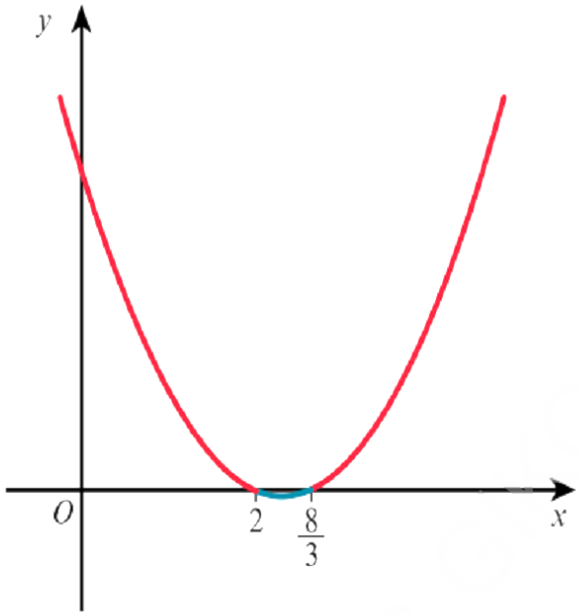
Then $x = 3$ 0.5

7.

Algebraic solution

$$|2x-5| > |3-x|$$
$$(2x-5)^2 > (3-x)^2$$
$$4x^2 - 20x + 25 > 9 - 6x + x^2$$
$$3x^2 - 14x + 16 > 0$$
$$(3x-8)(x-2) > 0$$

Critical values are 2 and $\frac{8}{3}$.



Hence $x < 2$ or $x > \frac{8}{3}$.

.....5

8. $2x^2 + 12x + 16 = 0$

$\leftrightarrow x^2 + 6x + 8 = 0$ 1

$\leftrightarrow (x+2)(x+4) = 0$ 1

$\leftrightarrow x+2=0, x+4=0$ 0.5

$\rightarrow x = -2, \quad x = -4$ 0.5

$S = \{-4, \quad -2\}$ 1

9. $f(x)=\dfrac{x^2-2}{x^2-8x+15}$

$x^2-8x+15\neq 0$ 1

$\leftrightarrow (x-3)(x-5)\neq 0$ 1

$\rightarrow x\neq 3, \quad x\neq 5$ 1

$Domf=\mathbb{R}-\{3,5\}$ 1

10. $f(x)=x^2-8$

let $f(x)=y$ 0.5

$y=x^2-8$ interchange y to x 0.5

$x=y^2-8$ 0.5

$y^2=x+8$ 0.5

$y=\sqrt{x+8}$ 0.5

$f^{-1}(x)=\sqrt{x+8}$ 0.5

11. $f(x)=\lim\limits_{x\to 2}\dfrac{x^3+x^2-4x-4}{x-2}$

$=\dfrac{(2)^3+(2)^2-4(2)-4}{2-2}=\dfrac{0}{0}$ *I.C.* 1

$=\lim\limits_{x\to 2}\dfrac{(x-2)(x+2)(x+1)}{x-2}$ 1

$=\lim\limits_{x\to 2}(x+2)(x+1)$ 1

$=(4)(3)=12$ 0.5

Then $\lim\limits_{x\to 2}\dfrac{x^3+x^2-4x-4}{x-2}=12$ 0.5

12. Calculating the first and second derivatives of $f(x)=x^2-4x+5$:

$f'(x)=2x-4$ and $f''(x)=2$ 0.5

The stationary points are the solutions of the equation $f'(x)=0$:

$f'(x)=0 \Leftrightarrow 2x-4=0 \Leftrightarrow x=2$.....0.5

$x=2$ is a stationary point. 0.5

To classify this point, we need to evaluate $f''(2)$.

.....0.5

In this case, $f''(2)=2>0$ so the function has a minimum at $x=2$

.....1

13. $\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 2 \\ -1 \end{pmatrix}+t\begin{pmatrix} 3 \\ -2 \end{pmatrix}=\begin{pmatrix} 2+3t \\ -1-2t \end{pmatrix}$ 0.5

Which gives

$x=2+3t$ 0.5

and

$y=-1-2t$ 0.5

Making t the subject of each equation gives

$t=\frac{x-2}{3}$ 0.5

and

$t=\frac{y+1}{-2}$, ...0.5

Hence

$\frac{x-2}{3}=\frac{y+1}{-2}$ 0.5

And so $-2x+4=3y+3$ 0.5

Or $2x+3y=1$ 0.5

14. If points are $(3,7)$, $(5,9)$

midpoint $=\frac{1}{2}(a_1+a_2,\quad b_1+b_2)$ 1

$=\frac{1}{2}(3+5,\quad 7+9)$ 1

$=\frac{1}{2}(8,\quad 16)$ 1

$=(4,\quad 8)$ 1

15. Event A : student is in the first year of study.

$p(A)=\frac{8}{20}=0.4$

.....1

$p(\overline{A})=1-p(A)=1-0.4=0.6$

.....1

The probability that the student is not the first year of study is 0.6

.....1

SECTION B: ATTEMPT ANY THREE QUESTIONS ONLY (45 MARKS)

a) $(-2)*(4*1)=(-2)*(4+1-4)$ ***.....1 mark***

$=-2*1$

$=-2+1+2$ 0.5marks

$=1$

$(-2*4)*1=(-2+4+8)*1$ .....0.5 marks

$=10*1$

$=10+1-10$

$=1$

$=10*1$

$=10+1-10$.....0.5marks

$=1$

As $(-2)*(4*1)=(-2*4)*1$ then the set $\mathbb{R}$ is associative under the operation $*$

.....0.5marks

b) Let e be the identity element under the operation $*$

We have $e*a=a=a*e$ .....0.5 marks

$e+a-ea=a=a+e-ea$ .....0.5 marks

$e(1-a)=0$.....0.5marks

$e=0$ provided $a\neq 1$.....0.5marks

c) Let $a_1*a=a*a_1=e=0$ 0.5marks

where a_1 is the inverse element of a then $a_1*a=a_1+a-a_1a=0$

.....0.5 marks

$a_1(1-a)=-a$

$a_1=-\dfrac{a}{1-a}$ 1mark

provided $a\neq 1$

The inverse of -3 is $\dfrac{3}{1+3}=\dfrac{3}{4}$.....1mark

17.

a) Existence condition:

$x^2-1\neq 0$ or $x\neq\pm 1$ ....1 mark

$Domf=\mathbb{R}\setminus\{\pm 1\}=]-\infty,-1[\cup]-1,1[\cup]1,+\infty[$ ....1 mark

b) $\lim\limits_{x\to -1}\dfrac{x^3}{x^2-1}=\infty$ .....*0.5 marks*

Roots of x^2-1 are $x=-1$ and $x=1$ ....0.5 marks

Sided limits:

x		-1		0		1	
$x+1$	$-$		0		$+$		$+$
x^2-1	$+$	0	$-$	$-$	0		$+$
$\frac{x^3}{x^2-1}$	$-$	∞	$+$	0	$-$	∞	$+$

$\lim\limits_{x\to -1^-}x=-\infty$ and $\lim\limits_{x\to -1^+}x=+\infty$

....0.5 marks

Thus, $V.A.\equiv x=-1$ ....0.5 marks

$\lim\limits_{x\to 1^-}x=-\infty$ and $\lim\limits_{x\to 1^+}x=+\infty$

Thus, $V.A.\equiv x=1$ ....0.5 marks

$\lim\limits_{x\to-\infty}\dfrac{x^3}{x^2-1}=\lim\limits_{x\to-\infty}\dfrac{x^2}{x}=\lim\limits_{x\to-\infty}x=-\infty$ ....0.5 marks

There is no horizontal asymptote $\lim\limits_{x\to+\infty}\dfrac{x^3}{x^2-1}=\lim\limits_{x\to+\infty}\dfrac{x^2}{x}=\lim\limits_{x\to+\infty}x=+\infty$ ....0.5 marks

There is no horizontal asymptote

Determining oblique asymptote(s):

$O.A.\equiv y=ax+b$ ....0.5 marks

$a=\lim\limits_{x\to\infty}\dfrac{f(x)}{x}=\lim\limits_{x\to\infty}\dfrac{x^3}{x^2-1}=\lim\limits_{x\to\infty}\dfrac{x^2}{x^2}=1\neq 0$ ....0.5 marks

$b=\lim\limits_{x\to\infty}\left[f(x)-ax\right]=\lim\limits_{x\to\infty}\dfrac{x^3}{x^2-1}-x$ ....0.5 marks

$$= \lim_{x\to\infty} \frac{x^3 - x^3 + x}{x^2 - 1}$$

$$= \lim_{x\to\infty} \frac{1}{x} = 0$$

Thus, $O.A. \equiv y = x$ ....0.5 marks

$\forall x \in Domf = \mathbb{R} \setminus \{\pm 1\}, -x \in Domf$ ....0.5 marks

c) $f(-x) = \frac{(-x)^3}{(-x)^2 - 1} = \frac{-x^3}{x^2 - 1}$ ....1 mark

$$= -\frac{x^3}{x^2 - 1} = -f(x)$$ ....0.5 marks

Therefore, f is odd function. .0.5 marks

d) For simplifying calculation $f(x) = \frac{x^3}{x^2 - 1} = x + \frac{x}{x^2 - 1}$ ....0.5 marks

$f'(x) = 1 + \frac{(x^2 - 1) - 2x^2}{(x^2 - 1)^2} = 1 - \frac{x^2 + 1}{(x^2 - 1)^2}$ ....0.5 marks

$$= \frac{x^4 - 2x^2 + 1 - x^2 - 1}{(x^2 - 1)^2}$$

$$= \frac{x^4 - 3x^2}{(x^2 - 1)^2} = \frac{x^2(x^2 - 3)}{(x^2 - 1)^2}$$ ....0.5 marks

x	$-\sqrt{3}$	-1	0	1	$\sqrt{3}$	
$x^2 - 3$	$+\ 0$	$-$	$-$	$-$	0	$+$
$f'(x)$	$+\ 0\ -$	$-$	0	$-$	$-\ 0$	$+$
$f(x)$	$-3\frac{\sqrt{3}}{2}$	$+\infty$	0	$+\infty$	$3\frac{\sqrt{3}}{2}$	$+\infty$
	$-\infty$	$-\infty$		$-\infty$		

To simply the calculation

$$f'(x)=1+\frac{(x^2-1)-2x^2}{(x^2-1)^2}=1-\frac{x^2+1}{(x^2-1)^2}$$

$$f''(x)=-\frac{2x(x^2-1)^2-2x(x^2+1)(x^2-1)}{(x^2-1)^4}$$ ....0.5 marks

$$=-2x(x^2-1)\frac{(x^2-1)-(x^2+1)}{(x^2-1)^4}$$ ....0.5 marks

$$=\frac{4x(x^2-1)}{(x^2-1)^4}=\frac{4x}{(x^2-1)^3},x\neq\pm1$$ ....0.5 marks

x		-1		0		1	
x^2-1	$+$	0	$-$		$-$	0	$+$
$x(x^2-1)$	$+$	0	$-$		$-$	0	$+$
$f''(x)$	$-$		$+$	0	$-$		$+$
$f(x)$	$\cap$	∞	$\cup$	0	$\cap$	∞	$\cup$

e) Table of variation ....2 marks

x	$-\sqrt{3}$	-1	0	1	$\sqrt{3}$
x^2-3	$+\ 0$	$-$	$-$	$-$	$0\ +$
$f'(x)$	$+\ 0\ -$	$-$	$0\ -$	$-$	$0\ +$
$f''(x)$	$-$	$+$	$0\ -$	$+$	
$f(x)$	$-\infty$ ↗ $-3\frac{\sqrt{3}}{2}$ ↘	$-\infty$ ↘ $+\infty$	$+\infty$ ↘ 0 ↘ $-\infty$	$-\infty$ ↘ $3\frac{\sqrt{3}}{2}$ ↗ $+\infty$	

$O.A. \equiv y = x$ $\quad V.A. \equiv x = -1$ $\quad V.A. \equiv x = 1$

18. a) $(A+B)^2 = \begin{pmatrix} 3 & -1 \\ 2 & 9 \end{pmatrix}\begin{pmatrix} 3 & -1 \\ 2 & 9 \end{pmatrix}$

.....1

$= \begin{pmatrix} 7 & -12 \\ 24 & 79 \end{pmatrix}$

.....2

$= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 7 & 10 \\ 15 & 22 \end{pmatrix}$

.....1

$$AB = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 2 & -3 \\ -1 & 5 \end{pmatrix} = \begin{pmatrix} 0 & 7 \\ 2 & 11 \end{pmatrix}$$

.....1

$$BA = \begin{pmatrix} 2 & -3 \\ -1 & 5 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} -7 & -8 \\ 14 & 18 \end{pmatrix}$$

.....1

$$B^2 = \begin{pmatrix} 2 & -3 \\ -1 & 5 \end{pmatrix}\begin{pmatrix} 2 & -3 \\ -1 & 5 \end{pmatrix} = \begin{pmatrix} 7 & -21 \\ -7 & 28 \end{pmatrix}$$

.....1

$$A^2 + 2AB + B^2 = \begin{pmatrix} 7 & 10 \\ 15 & 22 \end{pmatrix} + 2\begin{pmatrix} 0 & 7 \\ 2 & 11 \end{pmatrix} + \begin{pmatrix} 7 & -21 \\ -7 & 28 \end{pmatrix}$$

.....1

$$= \begin{pmatrix} 14 & 3 \\ 12 & 72 \end{pmatrix}$$

.....2

b) Thus $(A+B)^2 \neq A^2 + 2AB + B^2$ because the multiplication of matrices is not commutative, so $AB \neq BA$

.....2

c) Therefore the correct expansion of $(A+B)^2 = A^2 + AB + BA + B^2$ is

$$\begin{pmatrix} 7 & -12 \\ 24 & 79 \end{pmatrix} = \begin{pmatrix} 7 & 10 \\ 15 & 22 \end{pmatrix} + \begin{pmatrix} 0 & 7 \\ 2 & 11 \end{pmatrix} + \begin{pmatrix} -7 & -8 \\ 14 & 18 \end{pmatrix} + \begin{pmatrix} 7 & -21 \\ -7 & 28 \end{pmatrix}$$

.....1

$$= \begin{pmatrix} 7 & -12 \\ 24 & 79 \end{pmatrix}$$ as required

.....2

19. a) Solution

x	f	x^2	fx	fx^2
3	2	9	6	18
4	3	16	12	48
5	5	25	25	125
6	1	36	6	36
7	2	49	14	98
8	2	84	16	168
9	6	81	54	486
$\sum$	**21**		**133**	**979**

.....1 mark for each row = 8 marks (considering the eight last rows)

b) the mean

$$\bar{x} = \frac{\sum fx}{n} = \frac{133}{21}$$ 1

$$\bar{x} = 6.333$$ 1

c) Variance

$$\sigma^2 = \frac{1}{n}\sum fx^2 - \bar{x}^2$$ 1

$$\sigma^2 = \frac{1}{21} \times 976 - (6.33)^2$$

.....1

$$\sigma^2 = 6.407$$ 1

d) Standard deviation

$$\sigma = \sqrt{\frac{1}{n}\sum fx^2 - \bar{x}^2} = \sqrt{6.4072}$$

.....1

$$\sigma = 2.53$$ 1

20. a) There is no restriction(Combination) 1

$n = C_5^{15} =$ **3003 ways to form the committee.** 2

b) There is restriction for boys and girls:

$n = C_3^6 \times C_2^9 = 20 \times 36 =$ **720 ways to form the committee** 2

c)There are more girls than boys:

	Girls	**Boys**	***n***
Case1:	3	2	$C_3^9 \times C_2^6 = 1260$ ways 3
Case2:	4	1	$C_4^9 \times C_1^6 = 756$ ways 3
Case3:	5	0	$C_5^9 \times C_0^6 = 126$ ways 3

The total number of ways to get more girls than boys:

$1260 + 756 + 126 =$ **2142ways to form the committee**.

.....1